IES NATIONAL CENTER FOR EDUCATION STATISTICS
Institute of Education Sciences

The Nation's Report Card

Algebra I and Geometry Curricula

Results From the 2005 High School Transcript Mathematics Curriculum Study

NCES 2013-451

U.S. DEPARTMENT OF EDUCATION

WHAT IS THE HIGH SCHOOL TRANSCRIPT STUDY?

The High School Transcript Study (HSTS) collects and analyzes transcripts from a representative sample of America's public and private high school graduates. The study is designed to inform the public about the types of courses that graduates take during high school, how many credits they earn, and their grade point averages (GPAs). The HSTS also explores the relationship between coursetaking patterns and student achievement, as measured by the National Assessment of Educational Progress (NAEP). High school transcript studies have been conducted periodically for nearly two decades, permitting the reporting of trends in coursetaking and GPA, as well as providing information about recent high school graduates. In addition to collecting transcripts, the HSTS collects student information such as gender, graduation status, race/ethnicity, and information about the schools studied.

WHAT IS THE NATION'S REPORT CARD™?

The Nation's Report Card™ informs the public about the academic achievement of elementary and secondary students in the United States. Report cards communicate the findings of the National Assessment of Educational Progress (NAEP), a continuing and nationally representative measure of achievement in various subjects over time.

Since 1969, NAEP assessments have been conducted periodically in reading, mathematics, science, writing, U.S. history, civics, geography, and other subjects. NAEP collects and reports information on student performance at the national, state, and local levels, making the assessment an integral part of our nation's evaluation of the condition and progress of education. Only academic achievement data and related background information are collected. The privacy of individual students and their families is protected.

NAEP is a congressionally authorized project of the National Center for Education Statistics (NCES) within the Institute of Education Sciences of the U.S. Department of Education. The Commissioner of Education Statistics is responsible for carrying out the NAEP project. The National Assessment Governing Board oversees and sets policy for NAEP.

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Executive Summary

The 2005 National Assessment of Educational Progress (NAEP) High School Transcript Study (HSTS) found that high school graduates in 2005 earned more mathematics credits, took higher level mathematics courses, and obtained higher grades in mathematics courses than in 1990. The report also noted that these improvements in students' academic records were not reflected in twelfth-grade NAEP mathematics and science scores. Why are improvements in student coursetaking not reflected in academic performance, such as higher NAEP scores?

The Mathematics Curriculum Study (MCS) explored the relationship between coursetaking and achievement by examining the content and challenge of two mathematics courses taught in the nation's public high schools—algebra I and geometry. Conducted in conjunction with the 2005 NAEP HSTS, the study used textbooks as an indirect measure of what was taught in classrooms, but not how it was taught. In other words, the textbook information is not used to measure classroom instruction. Textbooks served as an indicator of the intended course curriculum (Schmidt, McKnight, and Raizen 1997). The chapter review questions in each textbook were used to identify the mathematics topics covered (or subject matter content) and the complexity of the exercises (or degree of cognitive challenge). Chapter review questions, and not the entire textbook, were coded because the questions have been found to be representative of the chapter content and complexity level in previous studies (Schmidt 2012). The study uses curriculum topics to describe the content of the mathematics courses and course levels to denote the content and complexity of the courses. The results are based on analyses of the curriculum topics and course levels developed from the textbook information, coursetaking data from the 2005 NAEP HSTS, and performance data from the twelfth-grade 2005 NAEP mathematics assessment. The study addresses three broad research questions:

1. What differences exist within the curricula of algebra I and geometry courses?
2. How accurately do school course titles and descriptions reflect the rigor of what is taught in algebra I and geometry courses compared to textbook content?
3. How do the curricula of algebra I and geometry courses relate to subsequent mathematics coursetaking patterns and NAEP performance?

In this report, curriculum topics, course levels, and grade 12 NAEP mathematics scale scores are used to describe the findings of the study. Curriculum topics are based on summaries of the textbook content that a school reported covering in an algebra I or geometry course. The six broad categories of curriculum topics used to describe the mathematics content found in both algebra I and geometry textbooks are: elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry, and other high school mathematics. **Table A** lists the content found within these curriculum topics.

TABLE A. Defining curriculum topics

Elementary and middle school mathematics	Basic arithmetic and pre-geometry
Introductory algebra	Pre-algebra, basic algebraic equations, and basic number theory
Advanced algebra	Advanced equations, basic functions, advanced functions, and advanced number theory
Two-dimensional geometry	Basic geometric concepts and properties of shapes
Advanced geometry	Three-dimensional geometry, coordinate geometry, and vector geometry
Other high school mathematics topics	Trigonometry, pre-calculus, statistics, validation and structuring, discrete mathematics, finite mathematics, and calculus

NOTE: Curriculum topics in this report are defined as the mathematics topics found in textbooks used in algebra I or geometry courses in high schools.

Course levels are rankings of courses that high school graduates took based on the combination of content and challenge of each course, as determined by the textbooks used. Courses were assigned only one course level. These rankings were developed separately for algebra I and geometry courses. For both courses, the three levels are beginner, intermediate, and rigorous (**table B**).

TABLE B. Defining course levels

Beginner	Covers more introductory material and less advanced material than an intermediate course.
Intermediate	Contains a balanced mix of both introductory and advanced material.
Rigorous	Covers more advanced material and less introductory material than an intermediate course.

NOTE: Course levels are used to describe the rank of high school algebra I and geometry courses, based on the textbooks they used. The rankings are based on the curriculum topics covered and the level of challenge posed to the students.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Results presented in this report are based on the 550 public schools and around 17,800 high school graduates selected for this study. This sample represents approximately two million public high school graduates from across the nation in 2005. Only high school graduates earning a regular or honors diploma are included in the analysis of this report, as is consistent with the reporting of the 2005 NAEP HSTS results. In addition, only graduates who took algebra I or geometry as high school courses were included in the study results. In 2005, 78 percent of all graduates took algebra I during high school and 20 percent of graduates took algebra I before entering high school. About 83 percent of all graduates took geometry during high school and 1.5 percent of graduates took geometry before entering high school.

The NAEP twelfth-grade mathematics results are reported as average scores on a scale of 0 to 300. The algebra and geometry scores are presented in the report to reflect performance on algebra I and geometry content, as opposed to overall mathematics performance. The MCS reports results using National Center for Education Statistics (NCES) statistical standards; findings from *t*-tests are reported based on a statistical significance level set at .05 without adjustments for multiple comparisons.

A few studies have analyzed textbook information and usage as a means to explain the apparent disconnect between coursetaking and achievement (Cogan, Schmidt, and Wiley 2001; Schiller et al. 2010; Tornroos 2005). The MCS adapted and built upon the methodology of these prior studies. See the Technical Notes of this report for a detailed description of the study methodology. While this study examined curriculum topics and course level of an algebra I or geometry course, it did not measure how well the curriculum was implemented in the classroom. In addition, only those graduates who took algebra I and geometry while in high school were included in the analyses. Therefore, students who took algebra I or geometry before entering high school were not included in the respective analyses because the textbook information was not collected. This limitation may be evident in the algebra I results, as those graduates who took the course in middle school were not included in the study results. Results from this study cannot be used to establish cause-and-effect relationships between mathematics textbooks and student mathematics coursetaking and performance.

Core content made up about two-thirds of graduates' algebra I and geometry courses.

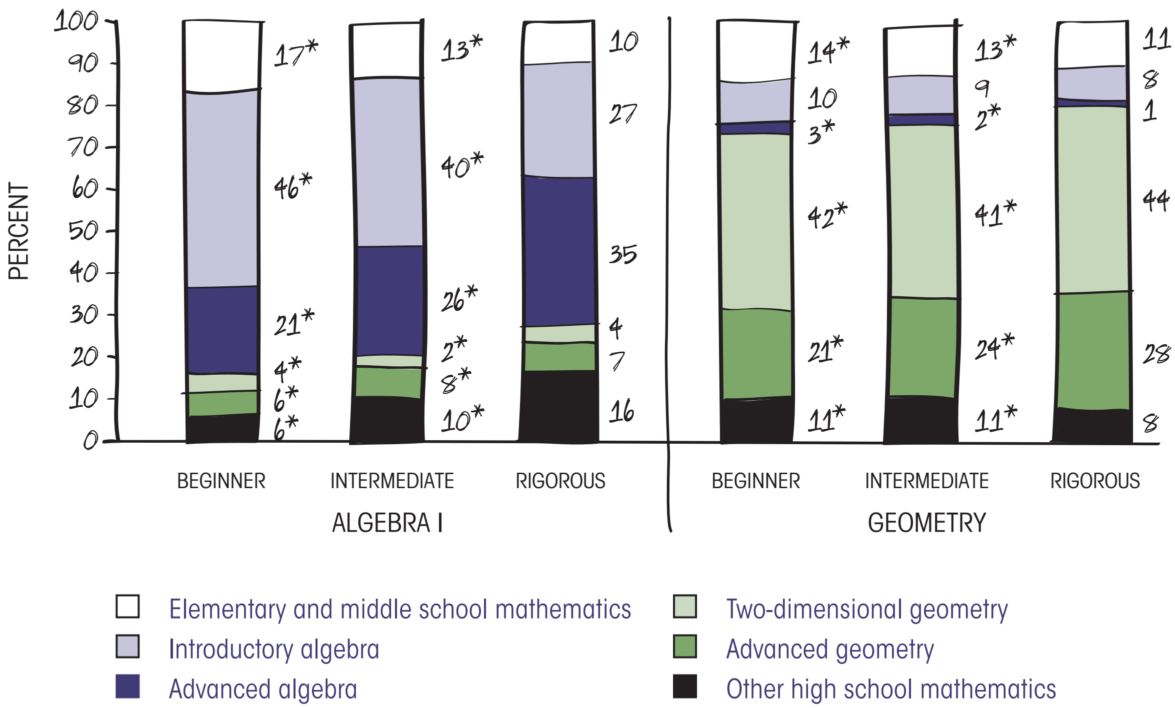
- In algebra I courses taken by high school graduates, about 65 percent of the material covered, on average, was devoted to algebra topics. About 35 percent of the material focused on elementary and middle school mathematics, geometry, and other high school mathematics topics typically taught in later mathematics courses.

- On average, about 66 percent of the material covered in geometry courses taken by high school graduates focused on geometry topics. About 34 percent covered elementary and middle school mathematics, algebra, and other high school mathematics topics.

Graduates' courses varied widely in the mathematics topics covered.

- About 17 percent of the course content of graduates' beginner algebra I courses focused on elementary and middle school mathematics topics, compared to 10 percent for graduates who took rigorous algebra I courses (**figure A**).
- For graduates who took rigorous algebra I courses, about 16 percent of the course content was other high school mathematics topics that are generally taught in higher-level courses, compared to 6 percent for graduates in beginner algebra I courses.
- About 14 percent of the course content of graduates' beginner geometry courses covered elementary and middle school topics, compared to 11 percent for graduates who took rigorous geometry courses.

FIGURE A. Percentage of content in graduates' algebra I and geometry courses, by course level and curriculum topic group: 2005



* Significantly different ($p < .05$) from rigorous.

NOTE: Details may not sum to total because of rounding.

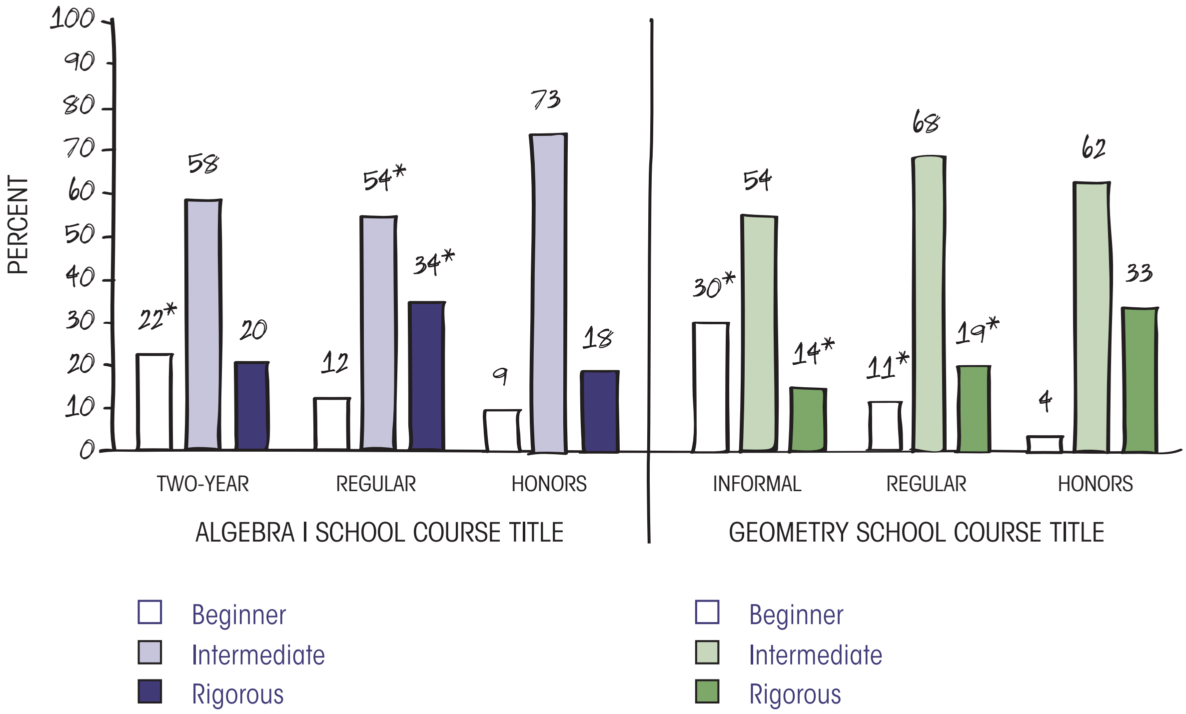
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

- For graduates who took rigorous geometry courses, 8 percent of their course content was other high school mathematics topics that are generally taught in higher level courses, compared to 11 percent for graduates who took beginner geometry courses.

School course titles often overstated course content and challenge.

- Approximately 73 percent of graduates who took an algebra I class labeled "honors" by their school received a curriculum ranked as an intermediate algebra I course (**figure B**).
- A higher percentage of graduates who took an algebra I class labeled "regular" by their school (34 percent) received a curriculum ranked as a rigorous algebra I course, compared to graduates who took an algebra I class labeled "honors" by their school (18 percent).

FIGURE B. Percentage of graduates in algebra I and geometry course levels, by school course title and course level: 2005



* Significantly different ($p < .05$) from honors.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated mathematics courses. "Two-year" algebra I is a course that is completed in two years. "Informal" geometry is a course that does not emphasize proofs.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

- Of the graduates who took a geometry course labeled "honors" by their school, approximately 33 percent received a curriculum ranked as rigorous geometry, whereas 62 percent received a curriculum ranked as intermediate geometry.

Few racial/ethnic differences by course level were found among subgroups who took similarly titled courses.

- Of the graduates who completed "two-year" algebra I courses, about 37 percent of Hispanic graduates received a curriculum equivalent to a beginner algebra I course, compared to 19 percent each of White and Black graduates.

- Of the graduates who completed "honors" geometry courses, about 37 percent of White graduates received a curriculum equivalent to a rigorous geometry course, compared to 17 percent of Hispanic and 21 percent of Black graduates.

- No racial/ethnic differences by course level were found among graduates who took classes labeled as "honors" algebra I. There were no measurable differences at any course level among White, Black, and Hispanic graduates who took either "informal" or "regular" geometry.

Fewer graduates who had beginner algebra I or geometry courses went on to complete advanced mathematics courses.

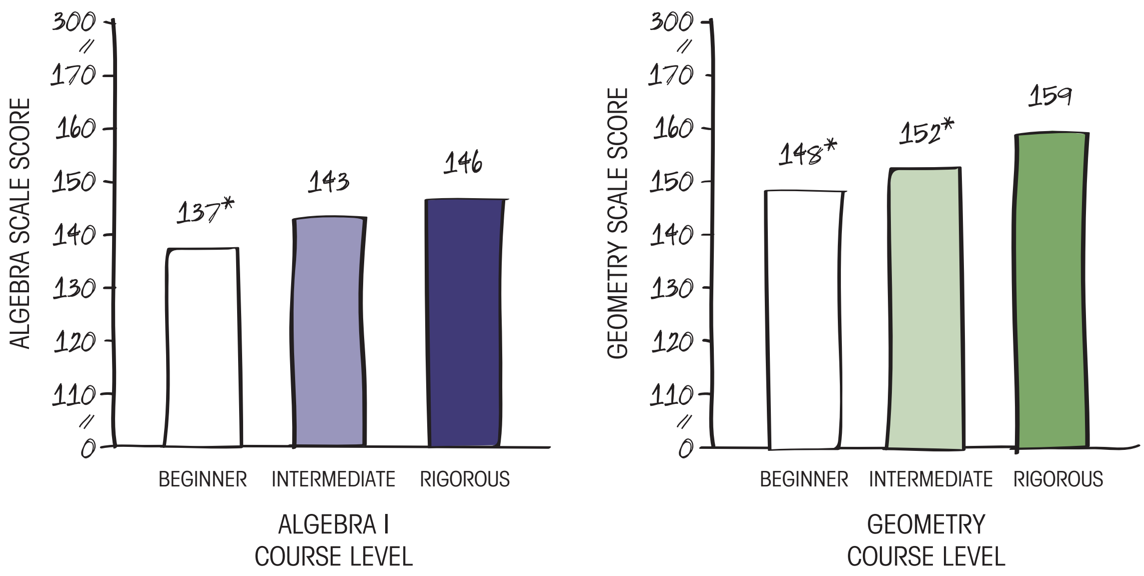
- About 60 percent of graduates who completed beginner algebra I courses went on to complete an algebra II course or higher as their highest level mathematics course, less than the 74 percent of graduates who had intermediate high school algebra I courses and 79 percent of graduates who had rigorous high school algebra I courses.

- Of the graduates who had a rigorous geometry course, about 50 percent took an advanced mathematics or calculus course as their highest mathematics course, comparatively higher than the 38 percent of graduates who had a beginner geometry course or the 42 percent who had an intermediate geometry course.

Graduates in rigorous algebra I and geometry courses scored higher on NAEP.

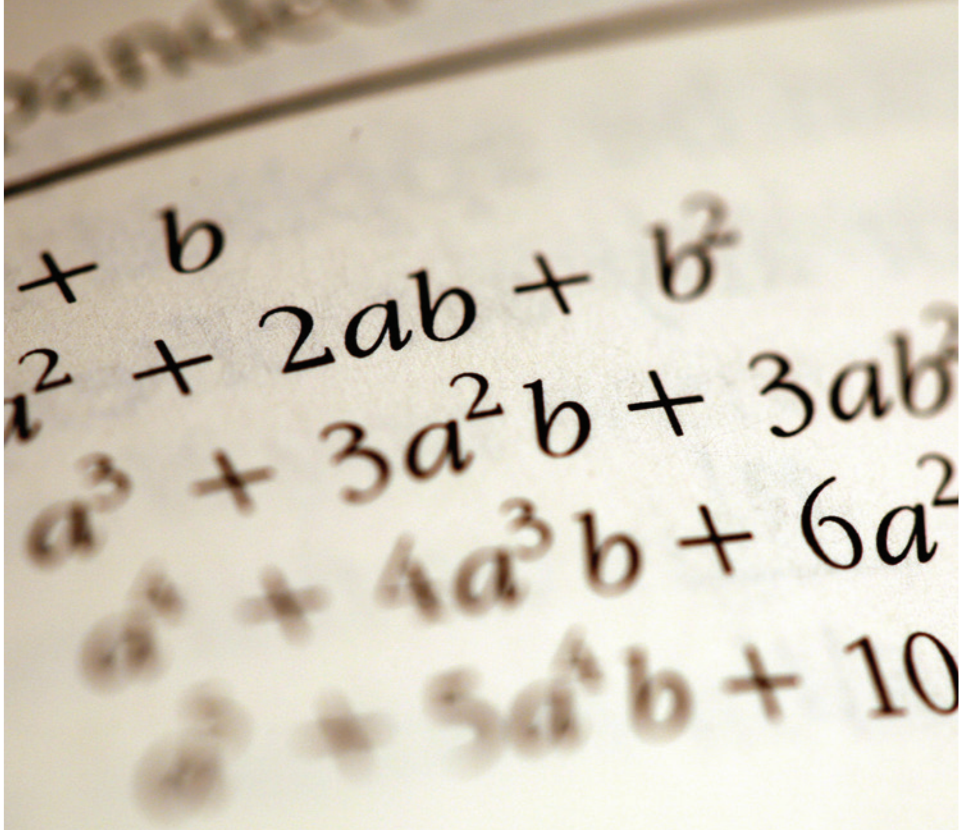
- Graduates who took rigorous algebra I courses obtained higher NAEP algebra scores (146) than graduates who took beginner algebra I courses (137) (**figure C**).
- Graduates who took rigorous geometry courses obtained higher NAEP geometry scores (159) than graduates who took beginner (148) or intermediate (152) courses.

FIGURE C. Average NAEP mathematics score of graduates in algebra I and geometry, by course level: 2005



* Significantly different ($p < .05$) from rigorous.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.



Introduction and Overview

The relationship between student coursetaking and academic performance has long been established. There is evidence that students who take advanced courses perform better academically than those students who do not take advanced courses (Shettle et al. 2007; Grigg, Donahue, and Dion 2007). Therefore, many reform efforts have focused on increasing the number of course credits required for high school graduation, including mathematics credits (Medrich et al. 1992; Chaney, Burgdorf, and Atash 1997; Stevenson and Schiller 1999). Results from the 2005 National Assessment of Educational Progress (NAEP) High School Transcript Study (HSTS) report (Shettle et al. 2007) found that 2005 high school graduates earned more credits, took a range of higher level courses, and earned higher grade point averages in mathematics than graduates in 1990.

The average number of credits in mathematics earned by 2005 graduates (3.8) was significantly higher than the average number of credits earned by graduates in 1990 (3.2). Graduates in 2005 earned a higher grade point average in mathematics courses (2.63) than graduates in 1990 (2.34). In addition, a higher percentage of graduates in 2005 than in 1990 completed a rigorous curriculum level. The rigorous curriculum level is used to report HSTS results (Shettle et al. 2007) and requires a graduate to take more advanced mathematics courses such as pre-calculus and calculus, advanced science courses, and more foreign language courses. Curriculum levels are based on the number of credits earned and the types of courses taken by graduates. Curriculum levels differ from the course levels discussed in this report.

The report also highlighted a lack of congruence between the HSTS and the NAEP. Improvements in student coursetaking that were shown in the 2005 NAEP HSTS report were not reflected in NAEP score trends (Shettle et al. 2007, p. 34).

For example, there was no measurable difference in the percentage of White and Black graduates who completed at least a midlevel curriculum in 2005. One of the requirements for achieving a midlevel curriculum is the completion of at least three years of mathematics courses, which include both algebra and geometry. The six-point percentage gap in 1990 between White and Black graduates completing at least a midlevel curriculum closed in 2005. However, performance gaps on the NAEP mathematics assessment remained (Shettle et al. 2007).

There are several plausible explanations for the lack of relationship between changes in high school coursetaking and NAEP score trends. The following are a few factors that might mitigate this relationship: changes in the population of students tested; declines in twelfth-graders motivation to do well on NAEP, a low-stakes assessment; and differences in course content (Shettle et al. 2007). Given all of these possible explanations, more in-depth analyses of these data are needed to understand the trends in student performance. The current study examines mathematics course content to further understand this relationship.

THE MATHEMATICS CURRICULUM STUDY

The Mathematics Curriculum Study (MCS) explores the relationship between student coursetaking and achievement by investigating the content and challenge of two core high school mathematics courses—algebra I and geometry. The study was conducted in conjunction with the 2005 NAEP HSTS.

Sample

The MCS brings together information from three sources—students, schools, and textbooks—to provide a more in-depth look at high school graduates' mathematics courses. During the 2005 NAEP HSTS data collection, 550 public schools provided textbook data for the study. The student sample included 17,800 graduates, which is representative of about 2 million graduates from across the nation. The analyses are limited to only those high school graduates who earned a regular or honors diploma, and completed an algebra I or geometry course during high school. In 2005, 78 percent of all graduates took algebra I during high school and 20 percent of graduates took algebra I before entering high school (see **table 1** on page 15). About 83 percent of all graduates took a geometry course during high school and 1.5 percent of all graduates took a geometry course before entering high school.

Methodology

Information from about 120 algebra I, geometry, and integrated mathematics textbooks was collected and coded for the study. Only those graduates who took an algebra I and/or geometry course that was linked to a textbook were included in the study analyses. Incorporating the textbook data with the transcript data, student and school demographic characteristics, and the NAEP mathematics assessment data allows for a comprehensive analysis of mathematics coursetaking and achievement.

The study's analyses are limited to textbook data linked to algebra I and geometry courses taken by public high school graduates. Public schools that did not offer algebra I and/or geometry courses, or comparable courses such as integrated mathematics, were not included in the study since none of the students in these schools could be connected to a textbook. Graduates who took algebra I during high school were included in the algebra analysis, while graduates who took geometry during high school were included in the geometry analysis.

The inclusion criteria for courses completed are independent of each other. For example, a graduate who took algebra I in eighth grade and geometry in ninth grade would be included in the geometry analyses, but not the algebra I analyses.

Textbook Coding

In this study, textbooks serve as indicators of the intended course curriculum as defined by Schmidt et al. (1997). The content of the textbook was used as an indirect measure of what was taught in classrooms (Tornroos 2005) because classroom instruction could not be measured in this study. That is, textbooks indicate the mathematics topics and types of skills a student will be exposed to in a course. Because textbooks are the main source of instructional material, they are used to measure what is taught in a course. About 120 textbooks were collected and analyzed for this study. The chapter review questions in each collected textbook, and not the entire textbook, were coded to determine two curriculum measures—the mathematics topic content and the level of cognitive challenge. The chapter review questions have been found to be representative of the chapter content and challenge level based on previous studies by Schmidt (2012). Both content and challenge were used in classifying graduates' classes into course levels. Content and challenge are not always directly related; that is, not all questions focused on low-level content have low degrees of challenge, and not all high-level content questions have high degrees of challenge. Coding textbooks at the chapter level allowed the study to distinguish between courses that covered the entire textbook and courses that only covered selected chapters from the textbook. Trained coders used a comprehensive framework of over 200 mathematics topics describing elementary and secondary education mathematics curriculum to identify the content covered in each textbook. (See **chart A1** in the Technical Notes for more details.) Information for chapters used in each course was aggregated by summing the mathematics topics covered and then connected to the graduates who took the courses.

The level of challenge for each textbook was determined by coding the chapter review questions, using about 25 major student performance expectations. Performance expectations are the activities or skills a student was expected to use to correctly answer a review question. The performance expectations for each chapter review question were ranked, and these ranks were averaged to create a level of cognitive challenge for the chapter. The overall cognitive challenge level for a course was aggregated by averaging the cognitive challenge level for the chapters covered within each textbook used in the course. These measures were then connected to the graduates who took the courses.

These two textbook curriculum measures were used to create two new measures that are used to describe the results of the study—curriculum topics and course levels. The curriculum data were analyzed along with coursetaking data from the 2005 NAEP HSTS and achievement data from the NAEP 2005 twelfth-grade mathematics assessment. For more detail on the textbook coding, refer to the 'Textbook Analyses' section of the Technical Notes.

Purpose

The MCS used measures of curricular content and challenge to address the following research questions:

1. What differences exist within the curricula of algebra I and geometry courses?
2. How accurately do school course titles and descriptions reflect the rigor of what is taught in algebra I and geometry courses compared to textbook content?
3. How do the curricula of algebra I and geometry courses relate to subsequent mathematics coursetaking patterns and NAEP performance?

Only a few studies have taken the approach of looking at textbook information and usage as a means to explain the lack of congruence between coursetaking and achievement (Cogan, Schmidt, and Wiley 2001; Schiller et al. 2010; Tornroos 2005). These three studies were limited by the number of textbooks examined, the number of schools participating, or the measures of achievement. Therefore, the present study builds on the methodology of prior studies by using a large national sample and the NAEP mathematics assessment data to measure achievement.

REPORTING THE RESULTS

In this report, curriculum topics, course levels, and NAEP mathematics scale scores are used to describe the findings of the study. The six categories of curriculum topics and three course levels referred to throughout this report were developed specifically for this study.

Curriculum Topics

Curriculum topics refer to broad categories of mathematics content topics that are covered in algebra I and geometry courses. Mathematics content topics were grouped by using a hierarchical structure of the curriculum framework and the grade level in which topics are introduced. (See the Technical Notes for more information on how the topics are aggregated.) Six main categories of curriculum topics were developed based on the content identified by the coding of textbook chapter review questions, as described in the previous section. Each is used to describe the mathematics content found in both algebra and geometry textbooks. These categories are as follows:

Elementary and middle school mathematics includes mathematics topics that are traditionally taught before a student takes an algebra I course. These topics include elements of basic arithmetic (e.g., addition, subtraction, fractions, and rounding) and pre-geometry (e.g., patterns, perimeter, area, and proportion).

Introductory algebra includes mathematics topics needed to understand the basics of algebra and provide the foundation for learning advanced algebra. These topics include pre-algebra, basic algebraic equations (e.g., algebraic expression, simple linear equations, and simple inequalities), and the basic elements of number theory (e.g., integers, absolute value, and rational numbers).

Advanced algebra includes mathematics topics that cover the more complex elements of algebra. These topics include advanced equations (e.g., quadratic equations, polynomial equations, and matrix solutions), basic functions (e.g., representation of relationships and functions, and graphing functions), advanced functions (e.g., functions of several variables and quadratic functions), and advanced number theory (e.g., real numbers, exponents, roots, radicals, and matrices).

Two-dimensional geometry includes mathematics topics that focus on basic linear and planar geometric concepts. Examples of topics in this category include basic geometric concepts (e.g., points, angles, parallelism, and perpendicularity) and the properties of shapes.

Advanced geometry includes mathematics topics that cover advanced geometric concepts such as three-dimensional geometry (e.g., three-dimensional shapes, conic sections), coordinate geometry (e.g., equations of lines, planes, and surfaces in space), and vector geometry (e.g., vectors, transformation, congruence, and similarity).

Other high school mathematics includes mathematics topics that are traditionally taught in courses taken after geometry and algebra II. Examples of topics in this category include trigonometry, pre-calculus, statistics (e.g., data representation and analysis, uncertainty and probability), validation and structuring (e.g., logic, set theory, and axioms), discrete mathematics (e.g., tree diagrams and binary arithmetic), finite mathematics, and calculus.

Course Levels

Course levels are rankings of students' algebra I and geometry coursework. They are based on both the curriculum topics covered and the level of challenge of high school graduates' courses, as determined by the content of their textbooks. Performance expectations were used to determine the degree of challenge.

Algebra I and geometry courses were grouped into three course levels—beginner, intermediate, and rigorous. Courses were assigned only one course level. While the rigorous course level is the highest level, it is not intended to denote an advanced course. The term "rigorous" is used to differentiate the course level from courses schools label "advanced." These levels are as follows:

Beginner level courses cover more introductory material and less advanced material than intermediate level courses.

Intermediate level courses contain a balance of both introductory and advanced material.

Rigorous level courses cover more advanced material and less introductory material than intermediate level courses.

Graduates who took integrated mathematics courses were not assigned a course level, but to a separate integrated mathematics category.

NAEP Scale Scores

The HSTS is conducted in conjunction with the NAEP. Therefore, the coursetaking patterns of the graduates can be examined relative to their educational achievement as measured by NAEP. Instead of looking at the overall mathematics scores, however, this study uses the content area scale scores—also called "subscale scores"—that focus on algebra and geometry as the achievement measure. These subscale scores correlate highly with the overall mathematics scores and are closely associated with the content taught in algebra I and geometry courses (http://nces.ed.gov/nationsreportcard/tdw/analysis/2004_2005/scaling_determination_correlations_math2005-conditional.asp). The 2005 NAEP twelfth-grade mathematics results—both overall and subscale scores—are reported as average scores on a scale of 0-300.

INTERPRETING THE RESULTS

The MCS presents subgroup comparisons. NCES uses widely accepted statistical standards when reporting results; findings from t-tests are reported based on statistical significance level set at .05 without adjustments for multiple comparisons (see the Technical Notes for more information). The symbol (*) is used in tables and figures to indicate that the percentage or performance of one group is significantly different from another group. Only those differences that are found to be statistically significant are discussed as higher or lower.

When scores are significantly different, then student performance is different. However, the MCS was not designed to identify the causes of these differences. More information about interpreting statistical significance can be found in the Technical Notes.

Although comparisons are made in students' performance based on demographic characteristics, the results cannot be used to establish cause-and-effect relationships between student characteristics and achievement. Many factors may influence student achievement, including educational policies and practices, available resources, and demographic characteristics of the student body.

High school graduates took algebra I courses before and during high school

It is important to keep in mind that the study analyses are limited to high school graduates who took algebra I or geometry courses while they were in high school. That is, only graduates who took algebra I during high school were included in the analysis sample for algebra I, just as only graduates who took geometry during high school were included in the analysis for geometry. Textbook information for courses taken before entering high school was not collected as a part of this study. Therefore, the lack of information on the content and challenge of algebra I courses taken before high school may impact the algebra I findings of this report. There are differences in the academic profiles of high school graduates who took algebra I before and during high school. **Table 1** below compares the academic and demographic characteristics of these two student groups. In 2005, one in five high school graduates had completed an algebra I course before entering high school. These graduates earned more total course credits, higher GPAs, and a higher overall score on the NAEP mathematics assessment than students who took algebra I in high school.

TABLE 1. Profiles of graduates who took algebra I before and during high school: 2005

	Before high school	During high school
Percent of all graduates	20	78
Student race/ethnicity		
Percent of White graduates	23	74
Percent of Black graduates	8	89
Percent of Hispanic graduates	10	87
Percent of Asian/Pacific Islander graduates	30	69
Coursetaking and performance		
Average total course credits earned	28.0*	26.6
Average credits earned in mathematics courses	4.2*	3.7
Average overall GPA	3.43*	2.88
Average GPA in mathematics courses	3.10*	2.52
Average overall NAEP mathematics score	182*	145

* Significantly different ($p < .05$) from graduates who took algebra I during high school.

NOTE: Data for graduates who did not take an algebra I course are not shown. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Understanding textbook coding

The four examples that follow illustrate how chapter review questions were coded to create the curriculum measures—curriculum topics and content levels—used to describe findings in this report. Chapter review questions from algebra I and geometry textbooks were coded for mathematics content and performance expectations, the latter measure being used to develop a degree of challenge. Both content and challenge were used in classifying graduates' classes into course levels. Content and challenge are not always directly related; that is, not all questions focused on low-level content have low degrees of challenge, and not all high-level content questions have high degrees of challenge. For detailed information on the curriculum topics and performance expectation coding, see the "Textbook Analyses" subsection on page 44 of the Technical Notes.

ALGEBRA I: ILLUSTRATIVE QUESTIONS

Question 1

Introductory algebra content with high degree of challenge

Curriculum topic: Basic algebraic equations.

Performance expectation: Communicating mathematical ideas and problem solving.

Question 1: Identify and correct any errors in the solution shown below.

Solve: $2x-(5x-2) = 4$

Solution: $2x-(5x-2) = 4$

$-3x-2 = 4$

$-3x = 6$

$x = -2$

Answer: An error occurs in the first step of the solution because the negative sign before the parentheses is not evenly distributed. After removing the parentheses, the first step should read $-3x + 2 = 4$. The next step should be $-3x = 2$. The answer is $x = -2/3$.

Question 2

Advanced algebra content with low degree of challenge

Curriculum topic: Advanced algebraic equations.

Performance expectation: Using algebraic procedures to manipulate formulas.

Question 2: At what rate would you have to invest to double your money in 20 years?

Compound interest formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Answer: 3.47 percent.

GEOMETRY: ILLUSTRATIVE QUESTIONS

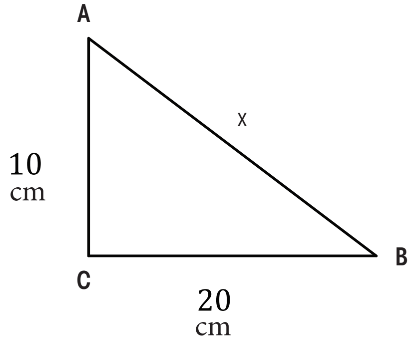
Question 3

Two-dimensional geometry content with low degree of challenge

Curriculum topic: Pythagorean Theorem.

Performance expectation: Recalling the Pythagorean Theorem and computation.

Question 3: In right triangle ABC, with the right angle at C, find x to the nearest tenth decimal place.



Answer: 22.4 cm

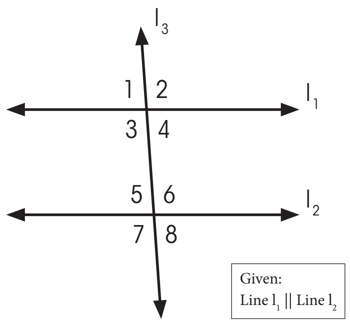
Question 4

Two-dimensional geometry content with high degree of challenge

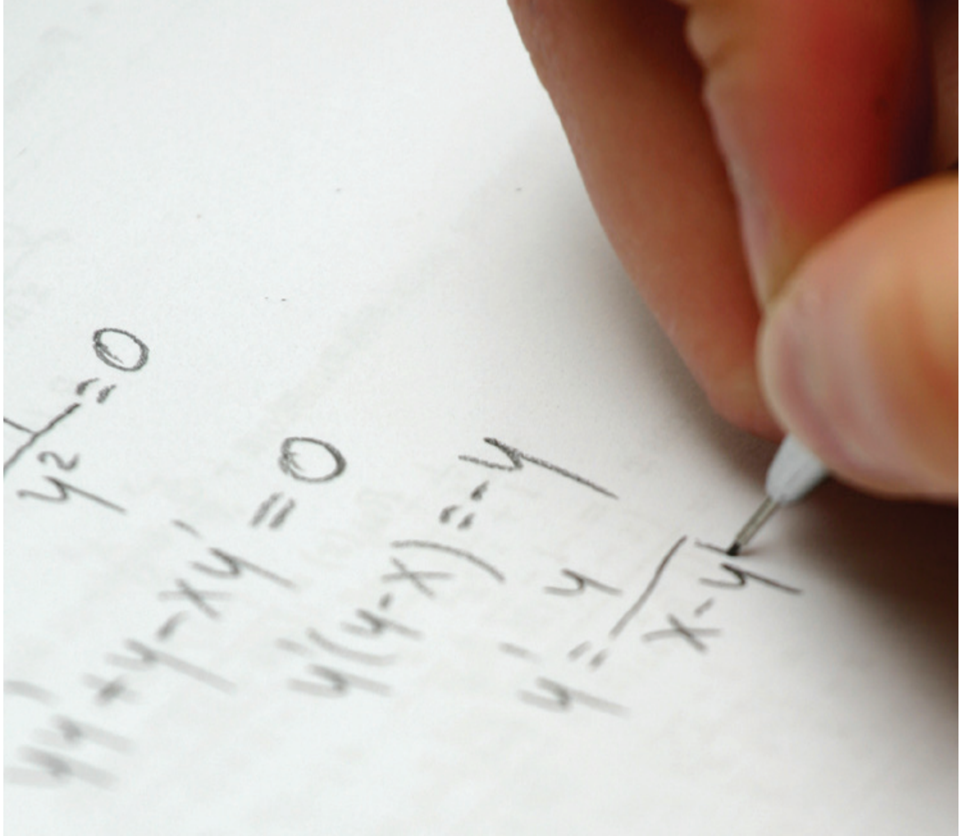
Curriculum topic: Angles and parallelism.

Performance expectation: Investigating and problem solving.

Question 4: Using the diagram below, if $m\angle 1 = 2x+30$ and $m\angle 6 = 3x+10$, where m denotes the measurement of an angle, find the measure of each angle.



Answer: $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8 = 86^o$
$m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7 = 94^o$



Mathematics Course Profiles

A profile of high school algebra I and geometry courses using six curriculum topics and three course levels is presented in this section of the report. Two-thirds of the content of algebra I and geometry courses focused on curriculum topics principal to the course, algebra I and geometry, respectively. The remaining one-third covered different mathematics topics. Across the nation, there was wide variation in the mathematics topics covered in graduates' algebra I courses, in particular in the percentage of content that is devoted to elementary and middle school mathematics. When disaggregated by race/ethnicity and course level, few measurable differences were found. Higher percentages of Hispanic and Asian/Pacific Islander graduates took courses ranked as beginner algebra I courses compared to White graduates.

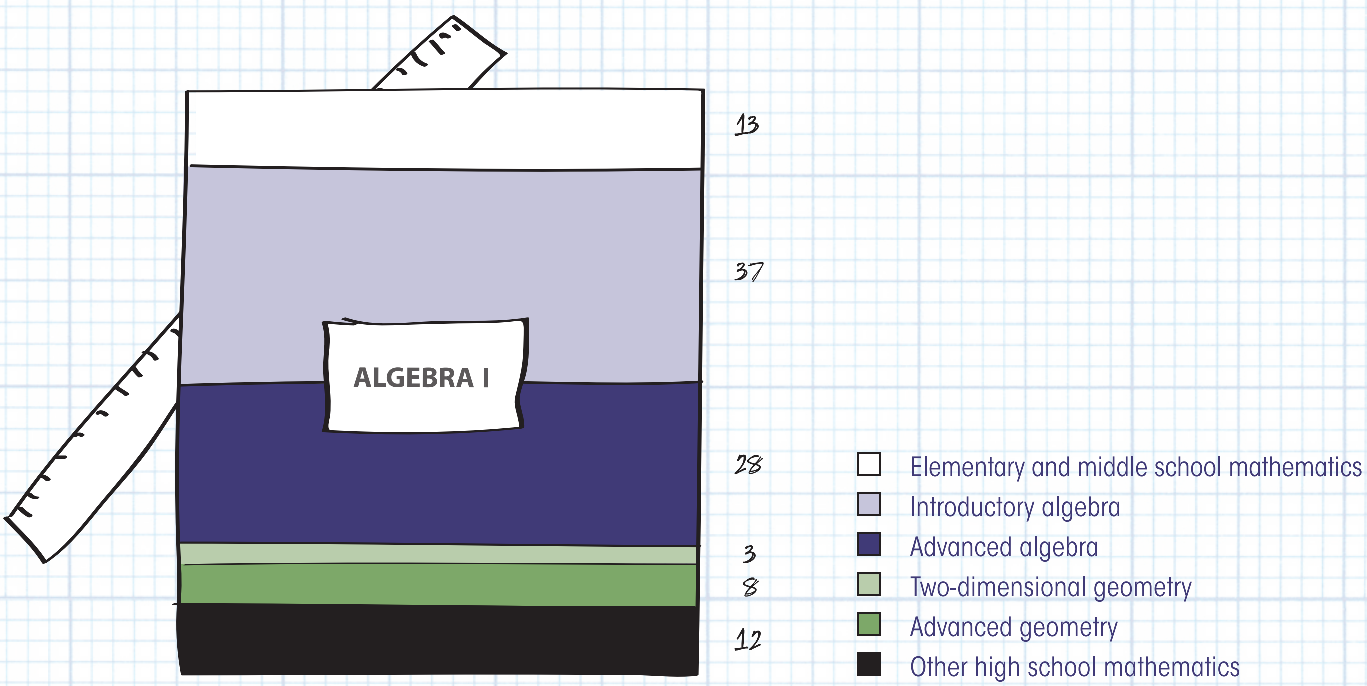
ALGEBRA I

What is algebra I? In general, algebra I courses focus on using symbols to express numbers and mathematical operations in equations, and manipulating mathematical expressions to solve for inequalities. Courses also concentrate on using functions to describe situations where one quantity determines another, such as rates of growth and decline. The mathematical operations that students are expected to perform become increasingly complex over the duration of a course. The following results describe high school algebra I courses, based on the textbooks used in the courses, using curriculum topics and course levels. It is important to keep in mind that textbook information was used as an indirect measure of the topics to be taught in a course, but does not reflect classroom instruction.

About two-thirds of an algebra I course consisted of algebra topics.

Figure 1 depicts the average mathematics content of high school algebra I courses by curriculum topics (see pages 12 and 50 for details). On average, 65 percent of a graduate's high school algebra I

FIGURE 1. Percentage of content of graduates' algebra I courses, by curriculum topic group: 2005



NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

course focused on algebra topics, including 37 percent on introductory algebra topics (e.g., pre-algebra and basic equations) and 28 percent on advanced algebra topics (e.g., advanced equations, basic and advanced functions, and advanced number theory). The remaining one-third of the content covered in a graduate's course was elementary and middle school mathematics (13 percent), two-dimensional geometry (3 percent), advanced geometry (8 percent), and other high school mathematics topics (12 percent) that are generally the focus of courses taken later in high school, like trigonometry, pre-calculus, and statistics.

Graduates who took rigorous courses had less review material than graduates who took beginner or intermediate courses.

Table 2 shows the percentage of content in algebra I courses taken by high school graduates broken down by course level. Graduates' algebra I courses varied widely in the mathematics topics covered. While all levels of algebra I courses contained some review material on elementary or middle school mathematics, on average, the percentage of this content was lower in rigorous level courses. For example, high school graduates in beginner level algebra I courses had, on average, 17 percent of their content focused on elementary and middle

TABLE 2. Percentage of content of graduates' algebra I courses, by course level and curriculum topic group: 2005

	Algebra I course levels			
Mathematics curriculum topic group	All levels	Beginner	Intermediate	Rigorous
Elementary and middle school mathematics	13	17*	13*	10
Introductory algebra	37	46*	40*	27
Pre-algebra	9	18*	9*	7
Basic equations	27	28*	31*	21
Advanced algebra	28	21*	26*	35
Advanced equations	15	12*	15	16
Basic functions	4	2*	2*	6
Advanced functions	2	#*	3*	2
Advanced number theory	8	6*	6*	11
Two-dimensional geometry	3	4*	2*	4
Advanced geometry	8	6*	8*	7
Other high school mathematics	12	6*	10*	16

Rounds to zero.

* Significantly different ($p < .05$) from rigorous.

NOTE: Details may not sum to total because of rounding. The categories that are indented are subcategories within the six broad curriculum topics: elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry and other high school mathematics.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

school mathematics, compared to 10 percent of the content for graduates in rigorous courses. The percentage of introductory algebra content followed the same pattern. About 46 percent of the content graduates covered in beginner courses was introductory algebra, compared to 27 percent of content for graduates in rigorous courses.

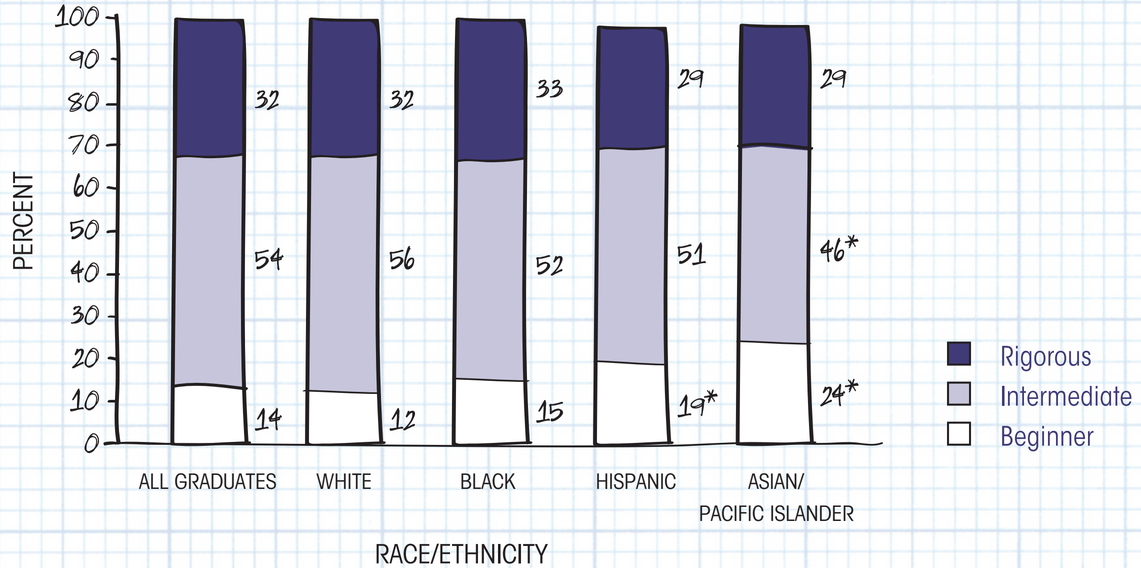
Conversely, the percentage of other high school mathematics topics introduced to high school graduates in algebra I courses was higher in intermediate and rigorous courses. Graduates in intermediate and rigorous courses received, on average, a larger percentage of content in other high school mathematics topics (10 and 16 percent, respectively) than graduates in beginner courses (6 percent). Graduates in rigorous courses had a larger percentage of advanced algebra topics (35 percent) than graduates in beginner courses (21 percent).

Most graduates, regardless of race/ethnicity, took an intermediate level algebra I course.

The percentage of high school graduates who took algebra I courses, by course level, is shown in **figure 2** for all graduates and by race/ethnicity. More than one-half (54 percent) of all graduates took an intermediate algebra I course. Approximately 14 percent had a beginner course and 32 percent took a rigorous course.

When comparing across racial/ethnic subgroups, there were no measurable differences among White, Black, and Hispanic graduates who took intermediate and rigorous algebra I courses. However, some differences were seen in the percentage of graduates who took beginner courses. A larger percentage of Hispanic (19 percent) and Asian/Pacific Islander (24 percent) graduates took a beginner algebra I course, compared to White graduates (12 percent). It is important to keep in mind the differences in percentages of students who took algebra I before entering high school across race/ethnicity. For example, 30 percent of Asian/Pacific Islander graduates took algebra I before entering high school (see **table 1**).

FIGURE 2. Percentage of graduates who took algebra I courses, by student race/ethnicity and course level: 2005



* Significantly different ($p < .05$) from White graduates.

NOTE: Details may not sum to total because of rounding. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

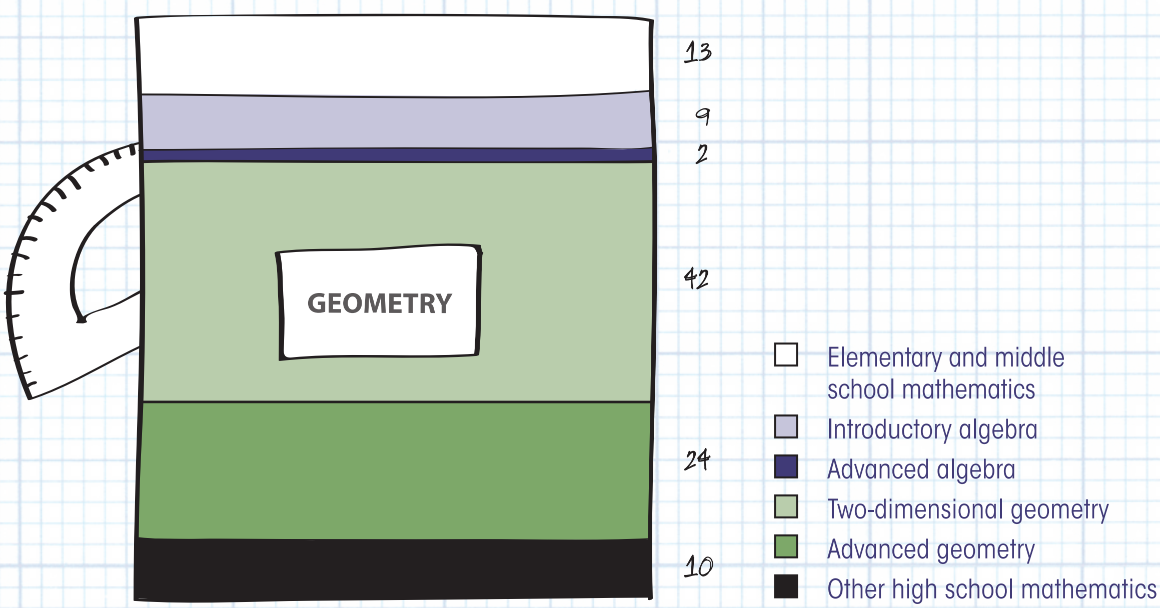
GEOMETRY

What is geometry? High school geometry courses are devoted to the formal analysis of two-dimensional shapes, the understanding of which can be applied in real-world contexts. Students are expected to use precise definitions and equations for analysis, which is more complex than the expectations of elementary and middle school classes. The results that follow describe high school geometry courses, based on the textbooks used in the course, using curriculum topics and course levels. It is important to keep in mind that textbook information was used as an indirect measure of the topics to be taught in a course, but does not reflect classroom instruction.

Geometry topics made up two-thirds of the content of geometry courses.

The mathematics content of high school geometry courses is shown in **figure 3**. On average, 66 percent of a graduate's geometry course focused on the core geometry topics of two-dimensional geometry (42 percent) and advanced geometry (24 percent), such as three-dimensional and coordinate geometry. The remaining one-third (34 percent) covered elementary and middle school mathematics review, algebra, and other high school mathematics topics.

FIGURE 3. Percentage of content of graduates' geometry courses, by curriculum topic group: 2005



NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

TABLE 3. Percentage of content of graduates' geometry courses, by course level and curriculum topic group: 2005

Mathematics curriculum topic group	Geometry course level			
	All levels	Beginner	Intermediate	Rigorous
Elementary and middle school mathematics	13	14*	13*	11
Pre-geometry	10	11*	10	10
Introductory algebra	9	10	9	8
Advanced algebra	2	3*	2*	1
Two-dimensional geometry	42	42*	41*	44
Advanced geometry	24	21*	24*	28
Three-dimensional geometry	6	7*	6	5
Coordinate geometry	4	3*	4*	7
Vector geometry	14	10*	14*	16
Other high school mathematics	10	11*	11*	8
Validation and structuring	6	7*	7*	5

* Significantly different ($p < .05$) from rigorous.

NOTE: Details may not sum to total because of rounding or omitted categories. The categories that are indented are subcategories within the six broad curriculum topics: elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry,
and other high school mathematics. Pre-geometry covers basic patterns, perimeter, area, volume, and proportionality.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Graduates' beginner geometry courses contained more review content; rigorous courses had more geometric content.

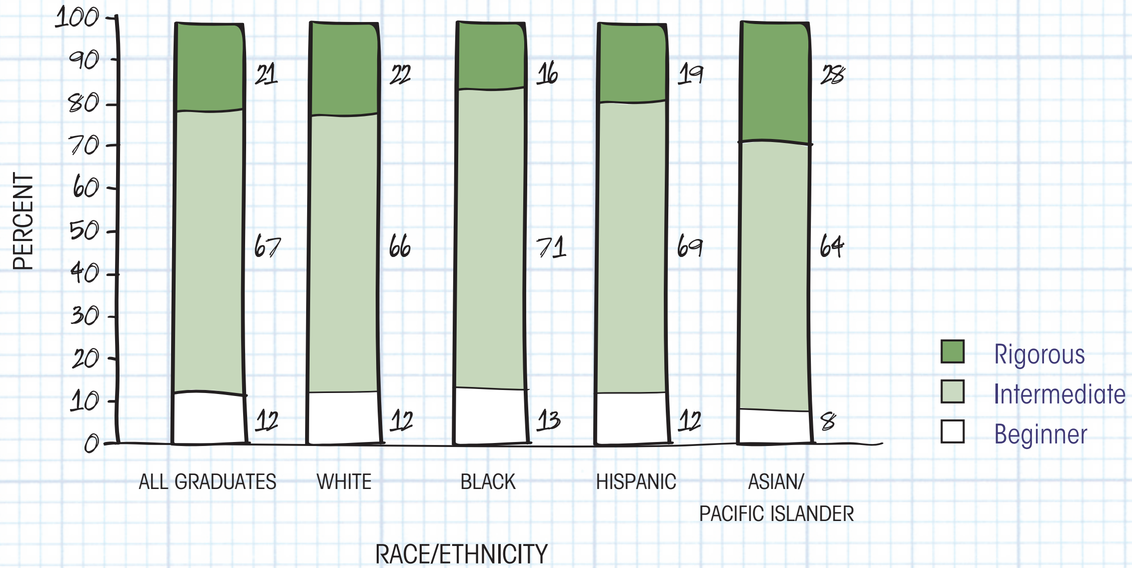
Table 3 shows the percentage of mathematics content of high school graduates' geometry courses broken down by course level. On average, graduates in beginner level geometry courses received a higher percentage of content in elementary and middle school mathematics topics than graduates in any other course level. In addition, graduates in beginner and intermediate level courses covered a higher average percentage of content in other high school mathematics topics (11 percent for both), compared to graduates in rigorous level courses (8 percent).

All graduates, regardless of the course level of their geometry class, had courses with a higher percentage of two-dimensional geometry topics than any other curriculum topic. However, graduates who had rigorous geometry courses received larger percentages of two-dimensional geometry and advanced geometry content than graduates in other course levels.

Most graduates took an intermediate geometry course.

The percentage of high school graduates who took a geometry course, broken down by course level and student race/ethnicity, is shown in **figure 4**. Approximately 12 percent of graduates took a beginner geometry course. Sixty-seven percent of graduates took an intermediate course, and 21 percent of graduates took a rigorous course. When compared to White graduates, there were no differences in the percentages of Black, Hispanic, or Asian/Pacific Islander graduates who took beginner, intermediate, or rigorous courses.

FIGURE 4. Percentage of graduates who took geometry courses, by student race/ethnicity and course level: 2005



* Significantly different (p < .05) from White graduates.

NOTE: Details may not sum to total because of rounding. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

INTEGRATED MATHEMATICS: AN ALTERNATIVE APPROACH TO TEACHING MATHEMATICS

Some states and school districts offer a different approach to teaching high school mathematics topics by integrating them into a single curriculum. Integrated mathematics (or unified mathematics) courses cover several mathematics topics or strands in one course, such as algebra, geometry, trigonometry, statistics, and analysis. Instead of separating these topics into individual courses, integrated mathematics programs interweave the topics taught. First-year integrated mathematics courses are generally taken at the same time most students take algebra I courses, while second-year integrated mathematics courses are taken when most students take geometry courses. There are textbooks designed specifically for integrated mathematics courses, although not all schools with integrated mathematics programs use them. Conversely, some schools adopt integrated mathematics textbooks for algebra I and geometry courses.

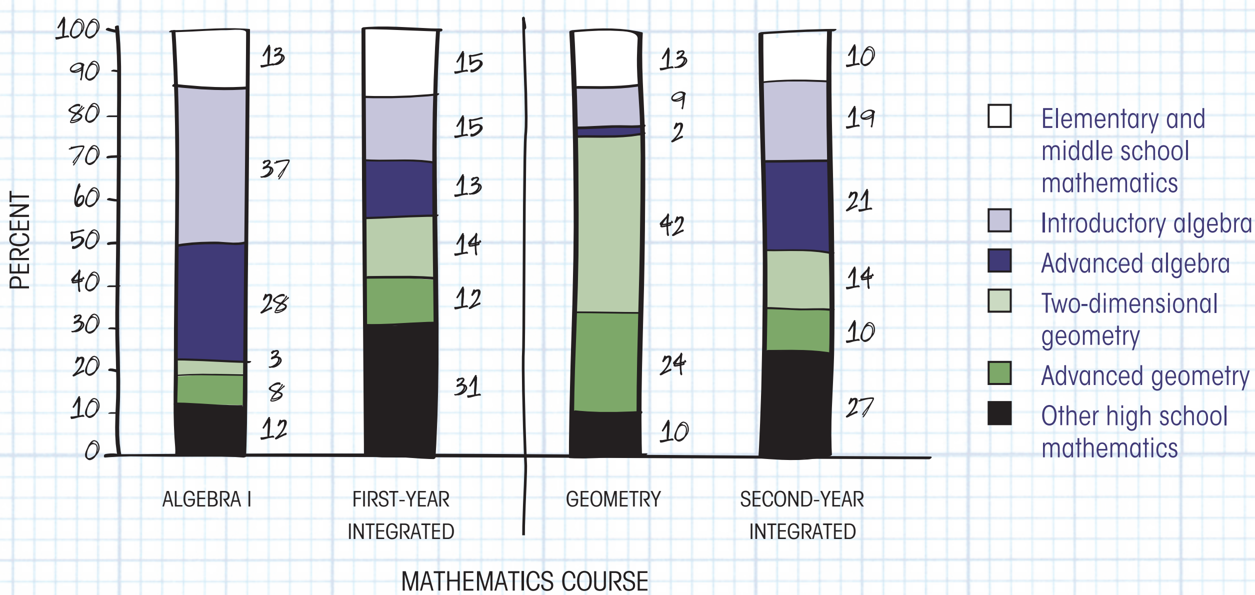
Only 6 percent of high school graduates completed a first-year integrated mathematics course, and 5 percent of graduates completed a second-year course. Due to the small number of graduates who took integrated mathematics courses, it was not possible to differentiate the course levels of integrated mathematics courses and meet reporting standards. Integrated mathematics courses were not ranked using course levels.

Advanced topics made up a quarter or more of integrated mathematics course content.

Figure 5 shows the mathematics profile of first- and second-year integrated mathematics courses taken by high school graduates. Graduates in first- and second-year integrated mathematics courses were exposed to more other high school mathematics topics, such as trigonometry, statistics, and calculus, than graduates in algebra I and geometry courses. These topics are typically the focus of courses taken after geometry.

Compared to traditional algebra I and geometry courses, integrated mathematics courses are not as focused on the core content of algebra or geometry. Whereas graduates in the average algebra I course had two-thirds of the course focused on algebra topics, graduates in first-year integrated mathematics courses had less than a third of the course devoted to this content (an average of 15 percent on introductory algebra and 13 percent on advanced algebra). Similarly, about a quarter of the content received by graduates in second-year integrated courses focused on

FIGURE 5. Percentage of content of graduates' algebra I, geometry, and integrated mathematics courses, by curriculum topic group: 2005



NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

either two-dimensional geometry (14 percent) or advanced geometry (10 percent), compared to an average of 66 percent for graduates in traditional geometry courses.

The average algebra and geometry content of high school graduates' first-year and second-year integrated mathematics courses is shown in **table 4**. Similar to traditional algebra I courses, the largest percentage of algebra content within graduates' first-year integrated course was basic equations (11 percent). The largest percentage of the geometry content for graduates in either a second-year integrated course or a traditional geometry course was two-dimensional geometry (14 percent and 42 percent, respectively).

TABLE 4. Percentage mathematics content of graduates' integrated mathematics courses, by curriculum topic group: 2005

Mathematics curriculum topic group	Integrated mathematics course	
	First-year course	Second-year course
Elementary and middle school mathematics	15	10
Pre-geometry	6	5
Introductory algebra	15	19
Pre-algebra	4	2
Basic equations	11	17
Advanced algebra	13	21
Advanced equations	4	7
Basic functions	4	3
Advanced functions	2	2
Advanced number theory	2	9
Two-dimensional geometry	14	14
Advanced geometry	12	10
Three-dimensional geometry	2	3
Coordinate geometry	5	4
Vector geometry	5	4
Other high school mathematics	31	27
Validation and structuring	1	4

NOTE: Details may not sum to total because of rounding or omitted categories. The categories that are indented are subcategories within the six broad curriculum topics: elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry, and other high school mathematics. Pre-geometry covers basic patterns, perimeter, area, volume, and proportionality.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.