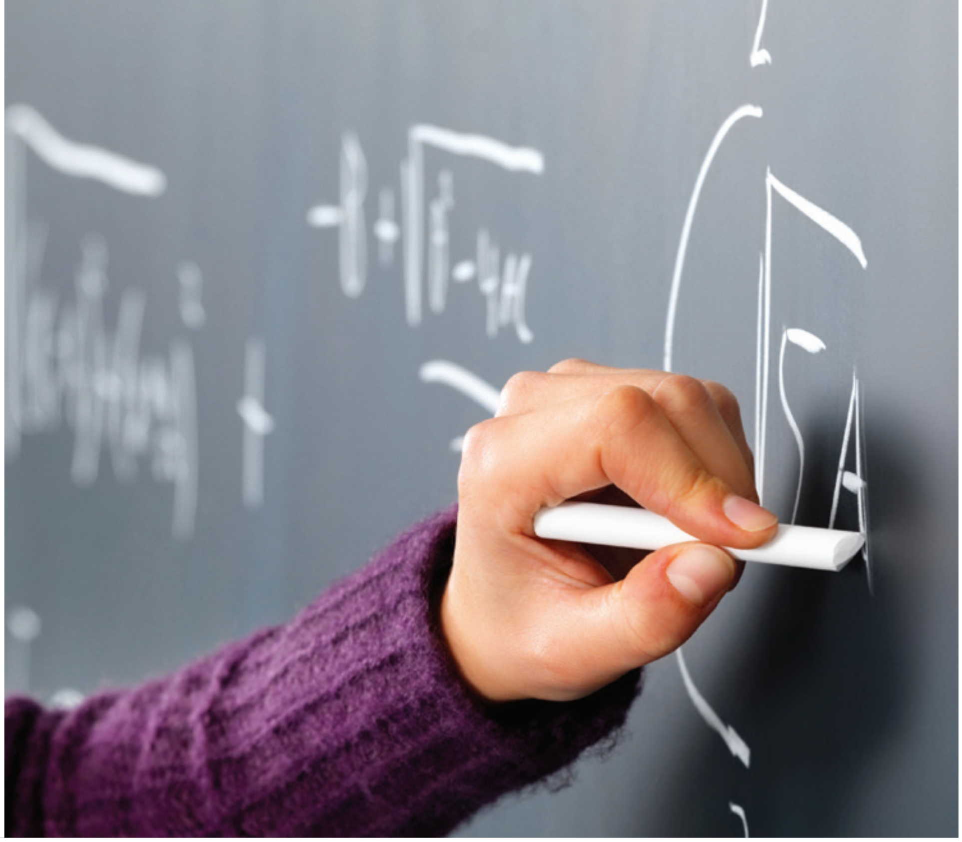


Comparison of School Courses

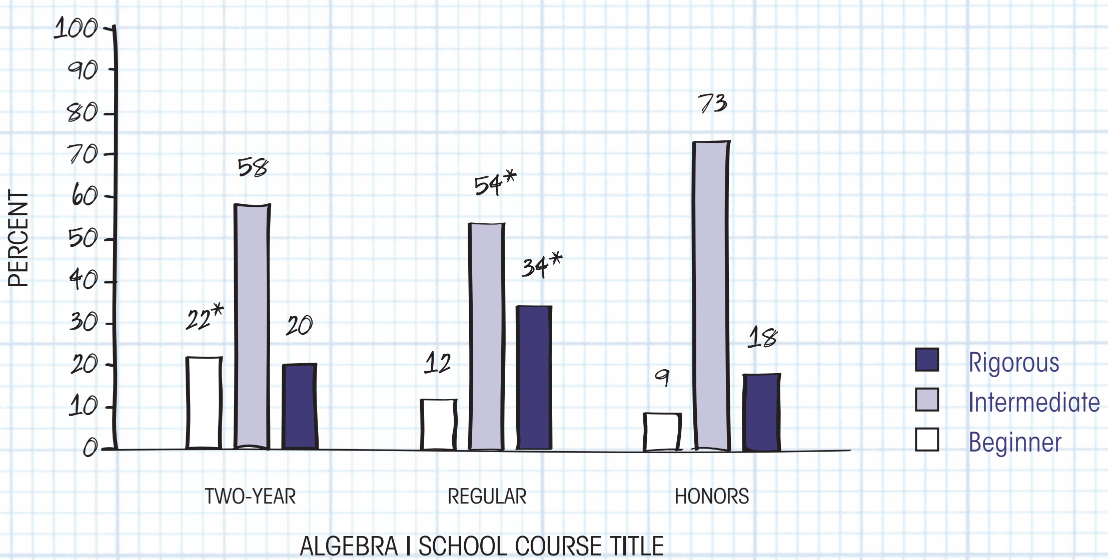
School course titles and descriptions are compared to course levels in this section of the report. The majority of algebra I and geometry classes were ranked as intermediate, regardless of the label given by the school. Seventy-three percent of graduates in classes the school labeled "honors" algebra I and 62 percent of graduates in classes the school labeled "honors" geometry were in courses ranked as intermediate. A larger percentage (37 percent) of White graduates in geometry classes labeled "honors" were enrolled in rigorous courses, compared to the percentage of Black and Hispanic graduates (21 and 17 percent, respectively) in similarly titled courses.

Most graduates' algebra I courses ranked at the intermediate level, regardless of the course title.

The percentages of graduates who took different types of algebra I courses, by school course descriptions and course levels, are shown in **figure 6**. Only 18 percent of high school graduates who took classes that schools labeled as "honors" algebra I were in courses ranked as rigorous, based on textbooks used by the schools. Most graduates who took classes schools labeled as "honors" algebra I (73 percent) were in intermediate courses. The percentage of graduates who took "honors" classes that were ranked as intermediate (73 percent) is larger than the percentage of graduates who took "regular" algebra I classes that were ranked intermediate (54 percent). About 9 percent of graduates in classes that schools labeled "honors" algebra I were in courses ranked as beginner.

Conversely, a larger percentage of graduates who took "two-year" algebra I classes (22 percent) were in courses ranked as beginner, compared to graduates who were in "honors" algebra I classes (9 percent). ("Two-year" algebra I is a course that is completed in two school years.)

FIGURE 6. Percentage of graduates who took algebra I classes, by school course title and course level: 2005



* Significantly different (p < .05) from honors.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated algebra I courses. "Two-year" algebra I is a course that is completed in two school years. "Honors" algebra I is a course that covers more advanced algebra topics and/or more in-depth analysis of algebra topics, including courses labeled honors, gifted and talented, and college preparatory.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

TABLE 5. Percentage of graduates who took algebra I classes, by course level, school course description, and student race/ethnicity: 2005

School course description and student race/ethnicity	Algebra I course level		
	Beginner	Intermediate	Rigorous
Two-year algebra I	22	58	20
White	19	60	21
Black	19	55	25
Hispanic	37*	53	11
Asian/Pacific Islander	‡	‡	‡
Regular algebra I	12	54	34
White	10	55	34
Black	14	52	34
Hispanic	17	52	32
Asian/Pacific Islander	22*	46	32
Honors algebra I	9	73	18
White	7	76	18
Black	15	67	18
Hispanic	10	56	35
Asian/Pacific Islander	‡	‡	‡

‡ Reporting standards not met.

* Significantly different ($p < .05$) from White graduates.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated algebra I courses. "Two-year" algebra I is a course that is completed in two school years. "Honors" algebra I is a course that covers more advanced algebra topics and/or more in-depth analysis of algebra topics, including courses labeled honors, gifted and talented, and college preparatory. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Few racial/ethnic differences by course level found among graduates in similarly titled algebra I courses.

Table 5 shows the course level breakdown of algebra I classes by school course descriptions and student race/ethnicity. Overall, few significant differences were found when comparing the percentages of White graduates who took algebra I courses by school course description and course level to Black, Hispanic, and Asian/Pacific Islander graduates who took similar courses, as noted in **table 5**. Approximately 37 percent of the Hispanic graduates in "two-year" algebra I classes were in beginner courses, compared to 19 percent each of White and Black graduates in similarly titled classes. When looking at graduates who were in courses titled "regular" algebra I classes, a larger percentage of Asian/Pacific Islander graduates, 22 percent, were in beginner courses compared to White graduates (10 percent). No racial/ethnic differences by course level were found among graduates who took classes labeled as "honors" algebra I.

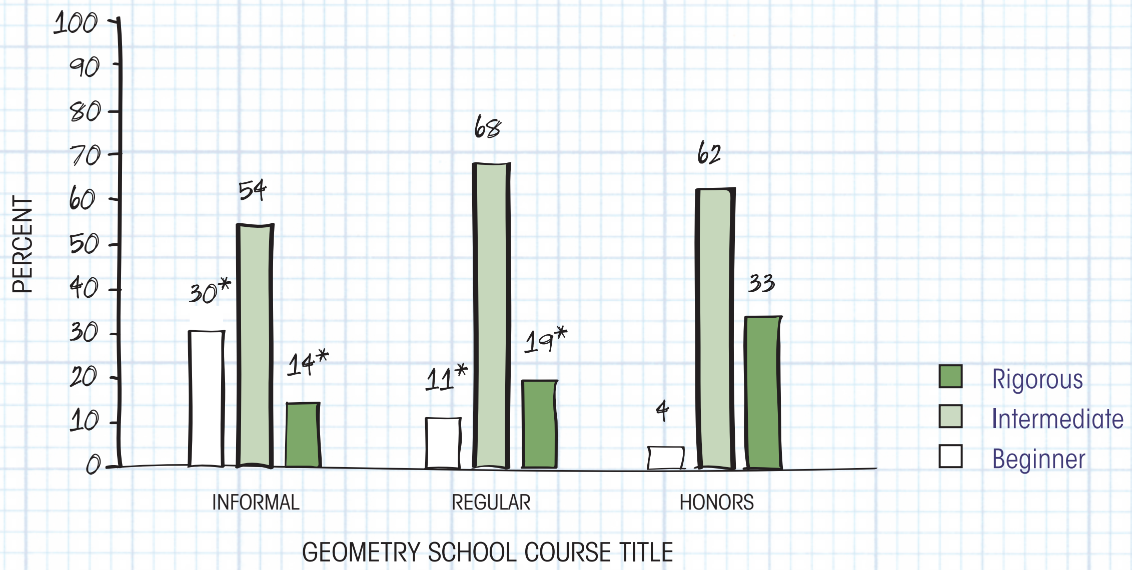
GEOMETRY

Most graduates received intermediate level geometry courses, regardless of course title.

The percentages of graduates who took different types of geometry courses, by school course descriptions and course levels, are shown in **figure** 7. One-half or more of the high school graduates enrolled in “informal,” “regular,” and “honors” geometry classes had an intermediate course. (“Informal” geometry is a course that does not emphasize proofs.) Fifty-four percent of graduates in classes that schools described as “informal” geometry, 68 percent of graduates in “regular” geometry, and 62 percent of graduates in “honors” geometry were in courses ranked as intermediate.

For those graduates in “honors” geometry classes, only 33 percent were in rigorous geometry courses. A larger percentage of graduates who took “honors” geometry classes had rigorous coursework, compared to graduates who took “informal” and “regular” geometry classes (14 and 19 percent, respectively). A larger percentage of graduates who took classes labeled “informal” geometry received coursework ranked as beginner level (30 percent), compared to graduates who took classes labeled “regular” or “honors” geometry (11 and 4 percent, respectively).

FIGURE 7. Percentage of graduates who took geometry classes, by school course title and course level: 2005



* Significantly different ($p < .05$) from honors.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated geometry courses. “Informal” geometry is a course that does not emphasize proofs. “Honors” geometry is a course that covers more advanced geometry topics and/or more in-depth analysis of geometry topics, including courses labeled honors, gifted and talented, and college preparatory.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Racial/ethnic differences by course level were evident only among graduates in courses titled "honors" geometry.

Table 6 shows the breakdown of geometry classes by course level, school course descriptions, and student race/ethnicity. For graduates in classes labeled "honors," 37 percent of White graduates had rigorous courses, compared to 21 percent of Black graduates and 17 percent of Hispanic graduates in similarly titled courses. One-half or more of graduates in each racial/ethnic subgroup who took "honors" geometry classes were in courses that were ranked as intermediate (57 percent of White, 73 percent of Black, 81 percent of Hispanic, and 63 percent of Asian/Pacific Islander graduates). A larger percentage of Black (73 percent) and Hispanic (81 percent) graduates who took "honors" geometry classes were in intermediate courses, compared to White graduates (57 percent). There were no measurable differences at any course level among White, Black, and Hispanic graduates who took either "informal" or "regular" geometry.

TABLE 6. Percentage of graduates who took geometry classes, by course level, school course description, and student race/ethnicity: 2005

School course description and student race/ethnicity	Geometry course level		
	Beginner	Intermediate	Rigorous
Informal geometry	30	54	14
White	29	53	17
Black	30	62	4
Hispanic	37	49	10
Asian/Pacific Islander	‡	‡	‡
Regular geometry	11	68	19
White	11	68	20
Black	12	72	16
Hispanic	12	69	19
Asian/Pacific Islander	9	64	27
Honors geometry	4	62	33
White	4	57	37
Black	6	73*	21*
Hispanic	2	81*	17*
Asian/Pacific Islander	2	63	35

‡ Reporting standards not met.

* Significantly different ($p < .05$) from White graduates.

NOTE: Details may not sum to total because of rounding and the use of integrated mathematics textbooks in nonintegrated geometry courses. "Informal" geometry is a course that does not emphasize proofs. "Honors" geometry is a course that covers more advanced geometry topics and/or more in-depth analysis of geometry topics, including courses labeled honors, gifted and talented, and college preparatory. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Coursework and Performance

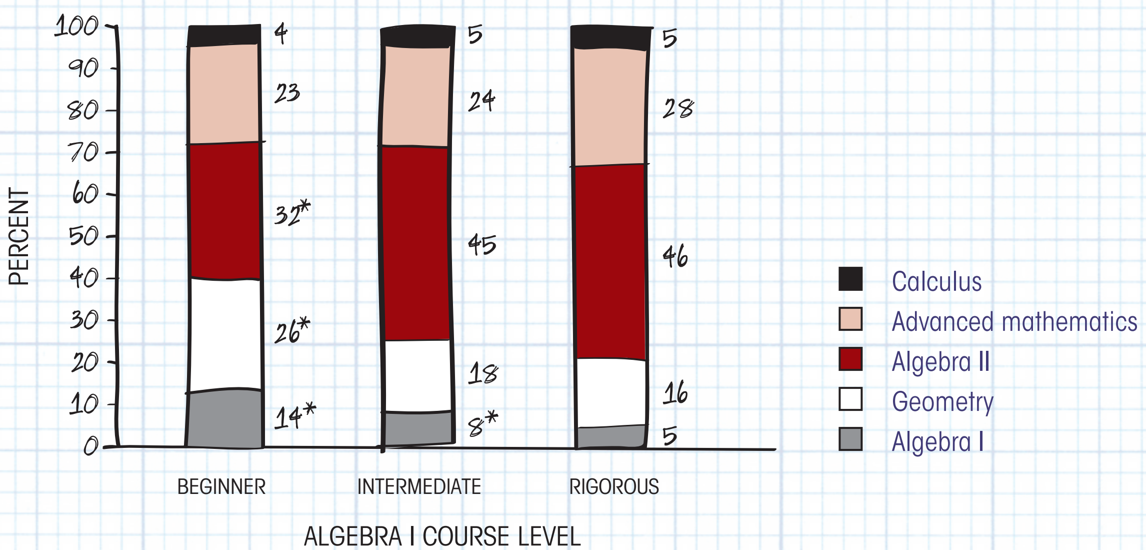
Graduates who took beginner, intermediate, or rigorous algebra I or geometry courses and their subsequent mathematics coursetaking and performance on the NAEP mathematics assessment are shown in this section of the report. The pattern for subsequent mathematics coursetaking is associated with both algebra I and geometry course level. Graduates who took rigorous algebra I courses had a higher average NAEP algebra score (146) than graduates who took beginner algebra I courses (137). Graduates who took rigorous geometry courses more often took a calculus course and achieved higher NAEP mathematics scores than graduates who took beginner or intermediate geometry courses.

COURSE LEVELS AND HIGHEST COURSE TAKEN

Graduates who took beginner algebra I courses more likely had algebra I or geometry as their highest mathematics course.

Figure 8 shows the highest level mathematics course taken by graduates given the course level of the algebra I class they took while in high school. A higher percentage of graduates who took a beginner algebra I course went on to have that course or a geometry course as their highest level mathematics course than graduates who took an intermediate or rigorous algebra I course. Of the high school graduates who had a beginner algebra I course, 14 percent had that class as their highest level mathematics course, which was higher than graduates who had an intermediate or a rigorous course. Similarly, a larger percentage of graduates (26 percent) who took a beginner algebra I course had geometry as their highest level mathematics course compared to graduates who had an intermediate (18 percent) or a rigorous course (16 percent). Fewer graduates who took beginner algebra I courses had algebra II as their highest level mathematics course taken than graduates who had intermediate or rigorous courses. About 32 percent of graduates who took a beginner algebra I course had algebra II as their highest mathematics course taken, compared with 45 percent of

FIGURE 8. Percentage distribution of highest level mathematics course taken by graduates, by algebra I course level: 2005

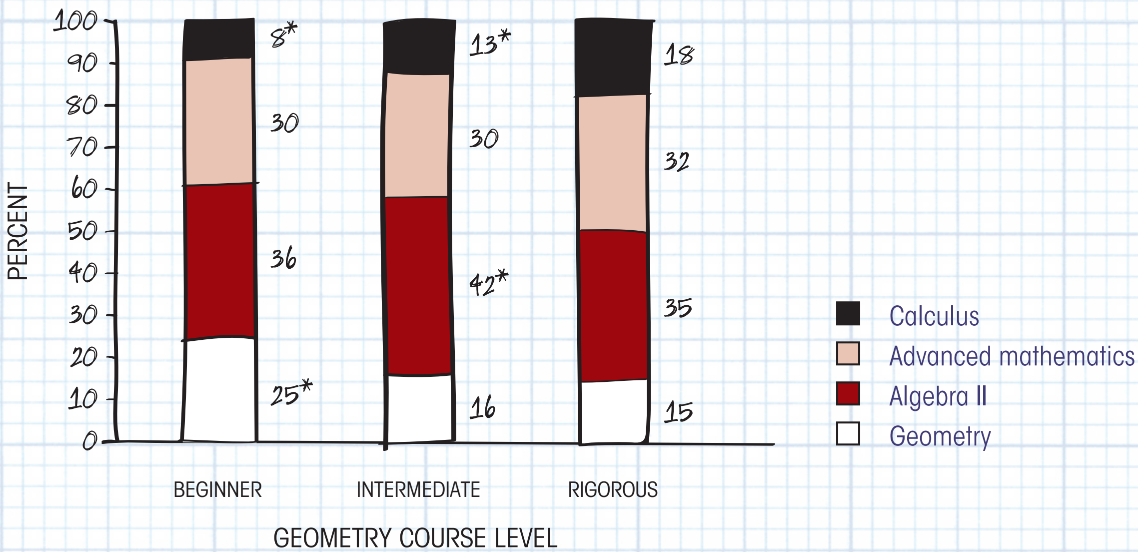


* Significantly different ($p < .05$) from rigorous.

NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

FIGURE 9. Percentage distribution of highest level mathematics course taken by graduates, by geometry course level: 2005



* Significantly different (p < .05) from rigorous.

NOTE: Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

graduates who took intermediate algebra I courses and 46 percent of graduates who took rigorous algebra I courses. There were no significant differences in the percentages of graduates who took an intermediate or rigorous algebra I course who went on to take a geometry course or higher as their highest level mathematics course.

A higher percentage of graduates who took rigorous geometry courses took advanced mathematics courses compared to graduates who took beginner and intermediate courses.

Figure 9 shows the highest level mathematics course taken by high school graduates given the course level of the geometry class they took while in high school. A larger percentage of graduates who received a rigorous geometry course took a calculus course in high school (18 percent) than those graduates who received beginner or intermediate courses (8 and 13 percent, respectively). However, a larger percentage of graduates who received a beginner geometry course had that course as their highest level mathematics course (25 percent) than those graduates who took an intermediate or rigorous geometry course (16 percent and 15 percent, respectively).

COURSE LEVELS AND NAEP ASSESSMENT SCORES

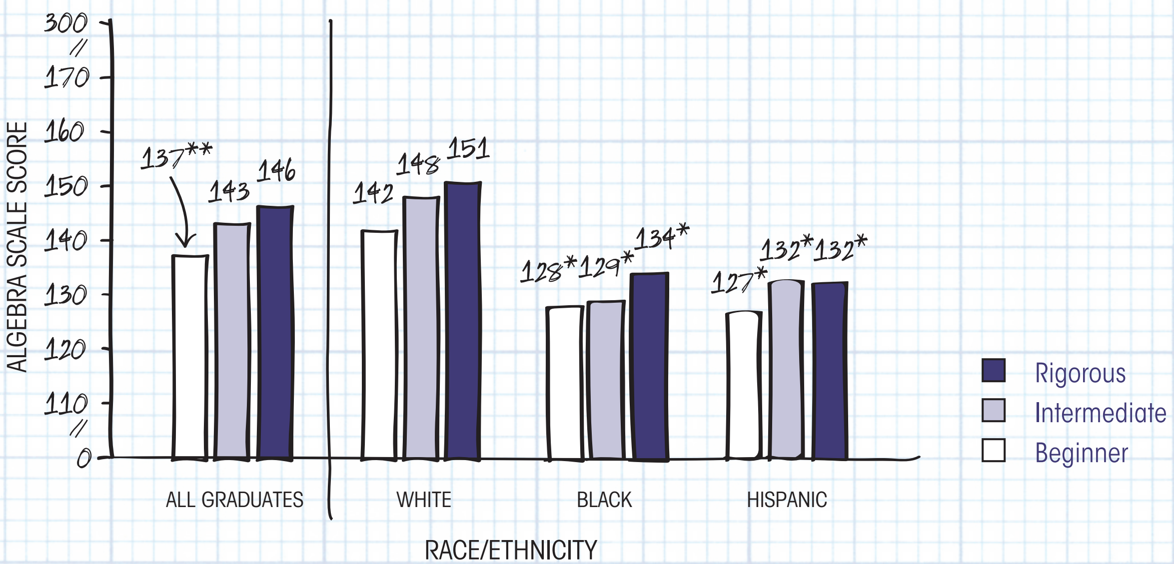
Graduates in rigorous algebra I courses performed better on NAEP than graduates in beginner algebra I courses.

The average NAEP algebra scores for 2005 high school graduates, by the course level of the algebra I classes taken and student race/ethnicity, are shown in **figure 10**. Graduates who took rigorous algebra I courses earned an average NAEP algebra score of 146. This average score was nine points higher than the average score of 137 earned by graduates who took beginner algebra I courses.

White graduates obtained higher scores across all algebra I course levels than Black or Hispanic graduates.

Score differences were also evident when the data were examined by student race/ethnicity. White graduates who took rigorous algebra I courses earned an average NAEP algebra score of 151, which was 17 points higher than the average scores obtained by Black graduates and 19 points higher than the average score obtained by Hispanic graduates who took rigorous algebra I courses. Significant achievement gaps were also

FIGURE 10. Average NAEP algebra score of graduates, by student race/ethnicity and algebra I course level: 2005



* Significantly different ($p < .05$) from corresponding White graduates.

** Significantly different ($p < .05$) from rigorous.

NOTE: Average NAEP algebra scale scores are shown. Asian/Pacific Islander graduates are included in the calculation of average NAEP algebra scores for "All Graduates" but are not reported separately because sample size does not meet reporting standards across course levels. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

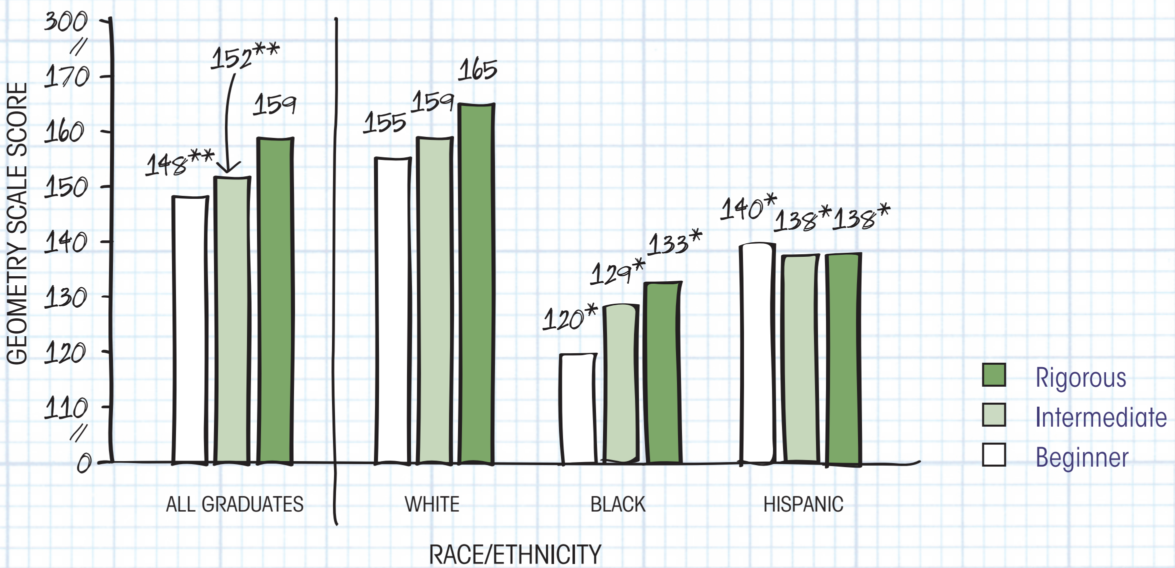
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

seen among White and Black graduates and White and Hispanic graduates who took beginner and intermediate algebra I courses.

Graduates in rigorous geometry courses also performed better on NAEP.

Figure 11 shows the average NAEP geometry score for 2005 high school graduates by the course level of the geometry classes they took and by student race/ethnicity. There was an eleven-point gap in the average geometry scores between graduates who took rigorous (159) and beginner (148) geometry courses. As was the case with algebra I, White graduates earned higher NAEP geometry scores than Black or Hispanic graduates, regardless of geometry course level.

FIGURE 11. Average NAEP geometry score of graduates, by student race/ethnicity and geometry course level: 2005



* Significantly different ($p < .05$) from corresponding White graduates.

** Significantly different ($p < .05$) from rigorous.

NOTE: Average NAEP geometry scale scores are shown. Asian/Pacific Islander graduates are included in the calculation of average NAEP geometry scores for "All Graduates" but are not reported separately because sample size does not meet reporting standards across course levels. Black includes African American, Hispanic includes Latino, and Asian/Pacific Islander includes Native Hawaiian. Race categories exclude Hispanic origin.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Technical Notes

Overview

The 2005 High School Transcript Study (HSTS) Mathematics Curriculum Study (MCS) brings together information from three sources—students, schools, and textbooks—to provide a more in-depth look at high school graduates' algebra I and geometry courses. The study used textbook data collected as part of the 2005 HSTS, which is associated with the National Assessment of Educational Progress (NAEP). Information from algebra I and geometry textbooks used by schools across the nation served as an indirect measure of the curriculum taught in these courses. Two mathematics curriculum measures—curriculum topics and course levels—were created from the textbook data and formed the basis of the study's results. These measures, along with the HSTS transcript data, student and school demographic data, and the NAEP 2005 twelfth-grade mathematics assessment data, were the data sources for the findings listed in this report.

A series of analyses were conducted for the textbooks collected, corresponding courses, and students who took the courses to create the curriculum topic and course level reporting measures. The textbook analyses, the first step, involved coding chapter review questions and creating chapter summary measures. Review questions for each chapter of a textbook, not the entire textbook, were coded for their curriculum topics (i.e., the mathematical topics covered) and performance expectations (i.e., the tasks required to answer the question). The chapter review questions have been found to be representative of the chapter content and challenge level (Schmidt 2012). Chapter summary measures were created to reflect the combined content of the chapter review questions for each textbook chapter.

The course analyses, the second step, involved generating course summary measures and creating categories of course content. The chapter summary measures of the textbook chapters used in each course were aggregated to create the course summary measures. A factor analysis of the course summaries was conducted to identify the patterns in the mathematics content coverage across courses, while a discriminant analysis was conducted to classify the courses into four content categories.

Lastly, the student analyses, the third step, involved generating student summary measures, classifying students' coursework into course levels, and defining broad categories of curriculum topics. The course summary measures were matched to students by the courses the students took, as listed on their transcripts, which allowed for the student summary measures to be created. Similar to the course analyses, a factor analysis of the student summaries was conducted to identify the patterns in the mathematics content coverage across students, while a discriminant analysis was conducted to classify the students' coursework into one of three course levels (beginner, intermediate, rigorous) and an integrated category. The student summary measures were also used to create six curriculum topic categories, which were also used as a reporting measure. The study methodology is summarized in this section of the report. The forthcoming Technical Report will provide full details of the study methodology.

Target Population and Analytic Sample

The HSTS has been conducted periodically since 1990. The MCS is a component of the HSTS administered in 2005. The target population for the 2005 study included students in public schools in the United States who were enrolled in the twelfth grade during the 2004-05 school year, who graduated in 2005 with a regular or honors diploma, and who took an algebra I course and/or geometry course (or an equivalent course, such as integrated mathematics) in high school. Private schools were not included in the analysis. Graduates with special education diplomas, certificates of attendance, or certificates of completion were excluded from the analysis.

Sampling

The 2005 NAEP HSTS sampling procedures were designed to achieve a nationally representative sample, including both public and private school graduates in the Class of 2005. Consistent with the 2005 NAEP national assessments, in the 2005 HSTS, students were selected for participation based on a stratified two-stage sampling plan. In the first stage of sampling, schools were selected, and students within schools were selected in the second stage. The weighting procedures for this study take into account the stratified sampling methods. The MCS uses graduates from the public school samples of the 2005 NAEP HSTS. The samples included the full student sample, which included approximately 37,900 public school students with transcript data, and a smaller sample, which only included the approximately 27,200 public school students who participated in the NAEP mathematics and science assessments. A full description of the sampling plan is beyond the scope of this Appendix. More sampling information about the 2005 NAEP HSTS can be found in The 2005 High School Transcript Study User's Guide and Technical Report (Shettle et al. 2008).

All public schools that participated in the 2005 NAEP HSTS were asked to fill out forms that identified the textbooks used for each mathematics course they offered. Schools that did not offer algebra I and/or geometry courses (or comparable courses such as integrated mathematics I and/or integrated mathematics II) or did not complete the textbook forms were not eligible for the MCS. High school graduates who did not take algebra I in high school were not included in the algebra I analysis. High school graduates who did not take geometry in high school were not included in the geometry analysis. About

17,800 public high school graduates from the 2005 NAEP HSTS were included in the MCS analyses. Around 12,500 graduates were part of the algebra I sample, while around 15,900 graduates were part of the geometry sample. The analyses for this report was limited to textbook data matched to algebra I and geometry courses. Approximately 5,700 students linked to the NAEP twelfth-grade mathematics assessment had textbook data matched to algebra I and/or geometry courses, of which 4,900 students were included in the MCS analyses.

As part of the 2009 NAEP HSTS, public schools were asked again to fill out forms that identified the textbooks used for each mathematics course offered. However, no further work has been conducted beyond the collection of these textbook lists. Therefore, no information from the 2009 data collection was included in this report.

School and Student Response Rates

Among the 640 public schools that participated in the 2005 NAEP HSTS, 550 schools had textbook data matched to their algebra I and/or geometry courses. A weighted school response rate was calculated by comparing the weighted percentages of schools that had textbook data with all 2005 NAEP HSTS participating public schools. Among all 2005 NAEP HSTS public schools, the weighted school response rate was about 85 percent.

As this report includes an analysis of NAEP twelfth-grade mathematics assessment scores, and since not all public schools in the HSTS participated in that assessment, a separate weighted school response rate was calculated based on the 2005 NAEP HSTS public schools linked to the NAEP 2005 twelfth-grade mathematics assessment. Of the 590 public schools that participated in both the 2005 NAEP HSTS and 2005 NAEP twelfth-grade mathematics assessment, 520 schools had textbook data matched to their algebra I and/or geometry courses. Among 2005 NAEP HSTS public schools that participated in the NAEP twelfth-grade mathematics assessment, the weighted school response rate was about 88 percent.

In addition to the school response rates, weighted student response rates were calculated by comparing the weighted percentages of students that were analyzed with textbook data with all eligible students. For the full student sample, eligible students were defined as public high school graduates who attended the schools that had textbook data that could be matched to algebra I and/or geometry courses. From the 550 public high schools with textbook data matched to algebra I and/or geometry courses, there were 21,100 eligible students, of which 17,800 students were included in the MCS analyses. Among these eligible students, the weighted student response rate was around 85 percent.

Similar to the school response rates, a separate weighted student response rate was calculated for students in schools linked to the NAEP 2005 twelfth-grade mathematics assessment. For the smaller NAEP-linked sample, eligible students were defined as public high school graduates who attended the schools that had textbook data that could be matched to algebra I and/or geometry courses and participated in the NAEP twelfth-grade mathematics assessment. From the 520 public high schools in the NAEP-linked sample that had textbook data matched to algebra I and/or geometry courses, there were 5,700 eligible students, of which 4,900 students were included in the MCS analyses. Among these eligible students, the weighted student response rate was around 87 percent.

Textbook Collection

About 120 textbooks—50 algebra I or first-year integrated mathematics textbooks and 70 geometry or second-year integrated mathematics textbooks—were selected for coding from the textbook lists provided by schools as being used in algebra I and geometry courses across the nation. The textbook lists included an indicator of whether the textbook was the main or supplemental textbook used for the mathematics course. The lists also included what chapters from the textbook were taught in the course.

Around half of the textbooks selected for coding had been coded in prior mathematics textbook studies conducted by the Center for Research in Mathematics and Science Education, formerly the U.S. Trends in International Mathematics and Science Study (TIMSS) National Research Center at Michigan State University (MSU). These previously coded textbooks were included in the sample because (1) schools were still using the textbooks; and (2) when new editions of textbooks were compared to previously coded editions, no changes were found in the chapter review questions. In both instances, the curriculum topics and performance expectations coded in prior studies were unchanged and, therefore, adopted for the current study.

To maximize student coverage, textbooks were selected for coding as part of this study based on the number of 2005 NAEP HSTS participants in courses using those textbooks. Textbooks used by only one or two schools were not sampled for coding to prevent possible disclosure of schools and students that participated in the NAEP HSTS and the MCS. Because information about the textbooks used by schools is publicly available, at least three schools had to have used an algebra I, geometry, or integrated mathematics textbook for it to be included in the current study.

Steps of the Analysis Process

The discussion of the series of analysis steps to conduct the MCS is broken into three sections: textbook analyses, course analyses, and student analyses. Figure A1 on the following pages displays an overview of the steps of the study analyses as a flow chart.

Steps of the Analysis Process

The Mathematics Curriculum Study used information from algebra I and geometry textbooks used in schools across the nation to characterize the mathematics content of these two courses. A series of analyses were conducted for the textbooks collected, the corresponding courses, and the students who took the courses to create the curriculum topic and course level measures that were used for reporting. The following flow chart illustrates the overall analysis steps and the text briefly describes what occurred in each step. A more detailed description of the analysis steps follows in the remaining pages of the Technical Notes.

STEP 1: Textbook Analysis

Detailed on pages 44-45

Code Chapter Review Questions

The Trends in International Mathematics and Science Study (TIMSS) Framework, consisting of over 200 mathematics topics and 20 performance expectation codes, was used to code each chapter review question in the textbooks.

Create Chapter Summary Measures

The assigned curriculum topic and performance expectation codes were used to create instructional measures, total content page counts and weighted topic content page counts, for each textbook chapter. The over 200 mathematics topics were aggregated to 32 groupings and the 20 performance expectation codes were ranked on a 4 point scale.

STEP 2: Course Analysis

Detailed on pages 45-48

Create Course Summary Measures

After the chapter review questions were coded and the information was summarized for textbook chapters, measures of total content page counts, weighted topic content page counts, and overall cognitive challenge were created for algebra I and geometry courses.

Course Factor Analysis

Factor analysis was used to identify distinct patterns of mathematics topics across courses. The 32 general topic groupings were aggregated to 17 and used as input for the factor analyses. A separate factor analysis was conducted for the algebra I and geometry courses. The resulting factors, five for algebra I and six for geometry, were used in the course discriminant analysis.

Course Discriminant Analysis

Discriminant function analysis was conducted to classify the courses into four distinct categories that identified the differences in curriculum content across the nation—low, medium, high, and integrated. The course summary measures of total content page counts, overall cognitive challenge ratings, and factors from course factor analysis were used as input. The resulting categories were used in the student discriminant analysis.

Link to Student Transcripts

To assist in creating the student summary measures, the course summary measures were matched to students through the algebra I and geometry courses on the students' transcripts.

STEP 3: Student Analysis

Detailed on pages 48-49

Create Student Summary Measures

After the course summaries were matched to the students who took the algebra I and geometry courses, measures of total content page counts, weighted content page counts, and overall cognitive challenge were created for students.

Student Factor Analysis

Factor analysis was used to identify the patterns of mathematics topics across students who took algebra I and geometry courses. The 32 general topic groupings were aggregated to 17 and used as input for the factor analyses. A separate factor analysis was conducted for the algebra I and geometry courses. The resulting factors, five for algebra I and six for geometry, were used in the student discriminant analysis.

Student Discriminant Analysis

Discriminant function analysis was conducted to classify student coursework into three distinct course levels that identified the differences in curriculum content and complexity across the nation—beginner, intermediate, and rigorous. An integrated category was also defined. The student summary measures of total content page counts, overall cognitive challenge ratings, and factors from student factor analysis were used as input.

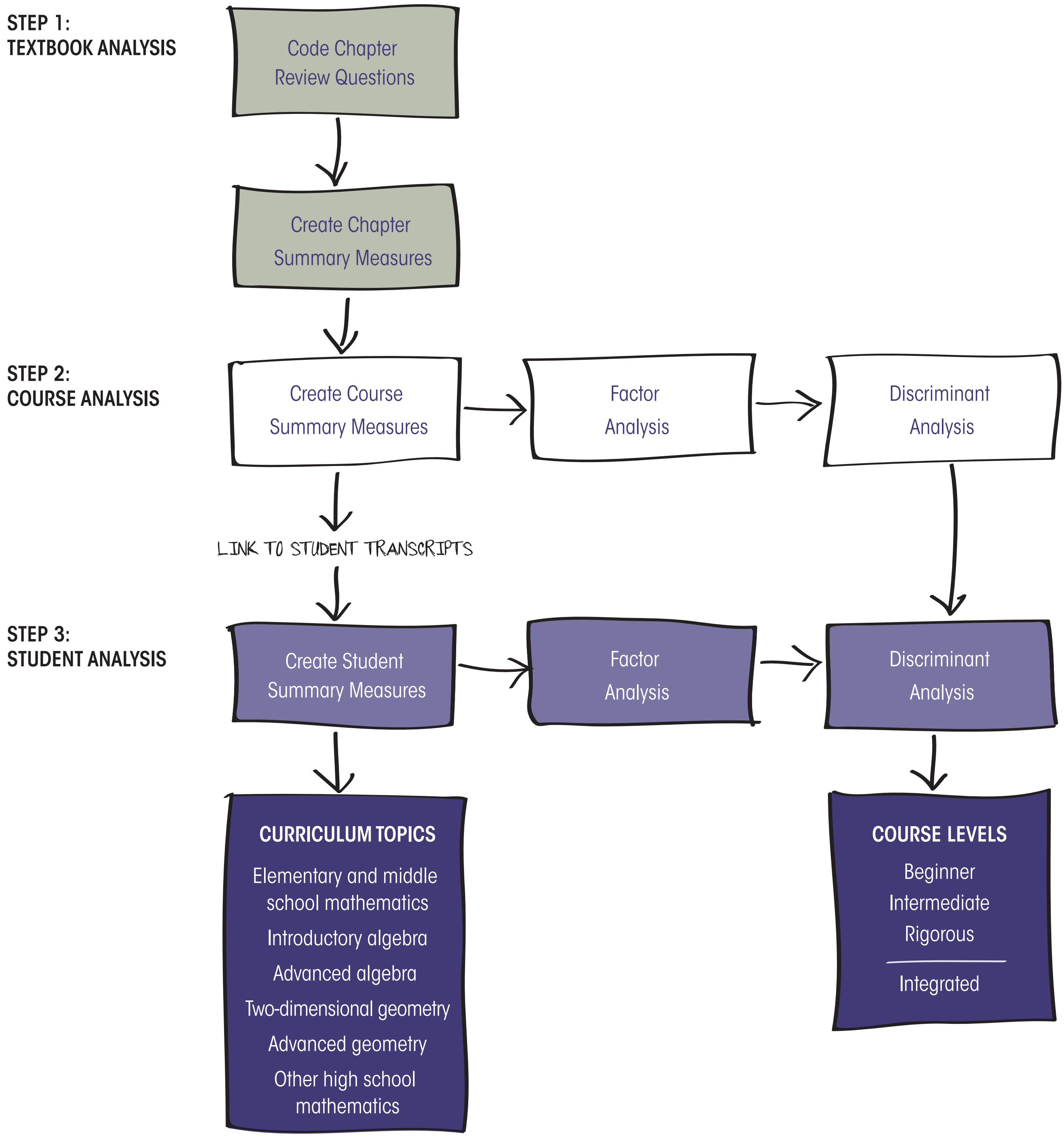
Define Course Levels

Based on the results of the student discriminant analysis, students' algebra I and geometry coursework was classified as one of three course levels—beginner, intermediate, and rigorous. An integrated category was also used for reporting.

Define Curriculum Topics

The 17 mathematics topic groupings used in the course and student factor analyses were collapsed into six broad categories for reporting students' algebra I and geometry coursework—elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry, and other high school mathematics.

FIGURE A1. Overall steps of the analysis process for the Mathematics Curriculum Study: 2005



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Textbook Analyses

The first step of the analyses for this study began with the textbooks. Two processes occurred during this step: coding of the chapter review questions, and creating chapter summary measures.

Code Chapter Review Questions

The textbook coding process for this study is based on frameworks developed for the Third International Mathematics and Science Study (TIMSS) (Robitaille et al. 1993; Schmidt, McKnight, and Raizen 1997) and was used in two other major national studies—the Longitudinal Study of American Youth (LSAY) and the Adolescent Health and Academic Achievement (AHAA) project (Muller et al. 2007; Schiller et al. 2008). The TIMSS framework provided the procedures for coding each review question found at the end of a textbook chapter for mathematics topics covered (curriculum topics) and the tasks required for students to answer the question (performance expectations). These data were aggregated to create summary measures for each textbook chapter, and the chapter summary measures were aggregated to create course summary measures.

Chapter review questions have been found to be representative of the chapter material based on previous work by Schmidt (2012); therefore, only chapter review questions were coded, not the entire textbook. By coding each review question at the end of each chapter, textbook contents were classified by the mathematics topics represented in the review question and the activities explicitly required to answer the question. Coding the activities as well as the topics provides a measure of textbook content that takes into account both contributions to student learning—the amount of instructional material and level of cognitive engagement (Gamoran et al. 1997). The topics and activities that were coded followed the TIMSS curriculum framework (Schmidt et al. 2001; Peak 1996). The procedures for coding the chapter review questions were the same procedures that were established for TIMSS. The procedures of recording topics and activities in the chapter review questions were intentionally designed for TIMSS to increase the reliability of coding across grade levels and nations. The procedures were described as low-inference (Schiller at al. 2010) because coders only record topics and tasks explicitly asked about in each chapter review question.

The coders used in the MCS were trained and supervised by the Center for Research in Mathematics and Science Education at MSU. The coders were required to have obtained graduate-level education in mathematics or mathematics education. The MCS used the same standardized coder training methods and quality control procedures used in the TIMSS (Schmidt, McKnight, and Raizen 1997) and AHAA (Schiller et al. 2008) studies.

Each potential coder for the MCS participated in a training session. During the training session, the coding supervisor introduced the coders to the TIMSS framework, gave instructions on how to code curriculum topics and performance expectations, and instructed the coders on how to complete the textbook coding forms. For hands-on coding practice, each potential coder was given an algebra I or geometry textbook to code during the training session. At the end of the training session, the coding supervisor reviewed each coder's work and provided feedback. Coders who demonstrated at least 80 percent accuracy in their textbook coding during the training session were eligible to serve as coders during the MCS coding session. The standard of 80 percent accuracy was the same standard that was set for the TIMSS study (Schmidt, McKnight, and Raizen 1997). Coders who did not meet this standard were given additional training and feedback. If these coders still could not meet the 80 percent accuracy threshold, they were not eligible to serve as coders during the MCS coding session. All coding information for chapter review questions that were coded by potential coders who were not retained for the MCS coding session was discarded and the questions were reassigned for recoding. During the MCS coding session, the coding supervisor also monitored the coders' work. Frequent random checks of their coding were conducted throughout the textbook coding process to ensure that the standard of 80 percent accuracy was maintained. Because the MCS uses previously coded textbooks from the LSAY and AHAA textbook studies, and coding accuracies for the textbooks MCS used cannot be disaggregated from the overall study accuracies, an overall coding accuracy rate for the MCS could not be determined. Inter-rater reliability was not measured for this study.

The TIMSS curriculum framework of mathematics topics used for the textbook coding is a comprehensive listing of topics taught in mathematics in elementary and secondary schools around the world (Robitaille et al. 1993). The framework was designed by a committee of international education researchers. The framework contains over 200 topic codes and 20 performance expectation codes (Robitaille et al. 1993). In the textbook coding process, up to five main topics and three supplementary topics could be recorded for each chapter review question. Up to three performance expectation codes could also be recorded. Each topic and performance expectation code was assigned a Boolean value to indicate whether the topic or code was present or absent for the question. Chart A1 lists the topic and performance expectation codes used in the study.

Since the AHAA study (Schiller et al. 2008), the TIMSS mathematics curriculum framework was updated to reflect the latest trends in mathematics curriculum. The framework was designed so that emerging trends in mathematics curriculum could be tracked (Robitaille et al. 1993). Changes made to the framework included adding new mathematics topics, expanding established topics, and re-ordering the algebra topics. Chart A1 reflects all of the changes made to the TIMSS mathematics curriculum framework since the AHAA study was conducted in 2004.

The following examples show how the illustrative textbook questions of the "Understanding textbook coding" section (found on pages 16 and 17 of this report) would have been coded using the TIMSS curriculum framework, including the index numbers of the topics and performance expectations from chart A1:

- Question 1 (page 16): The topic code for this example is "Linear equations and their formal (closed) solutions" (1.6.2.5). Performance expectation codes for this example are (a) "Performing routine procedures" (2.2.2); (b) "Critiquing" (2.5.4); and (c) "Verifying" (2.3.5).

- Question 2 (page 16): The topic codes for this example are (a) "Logarithmic and exponential equations and their solutions" (1.6.2.9); (b) "Growth and decay" (1.8.2.1); and (c) "Substituting into or rearranging formulas" (1.6.2.15). The performance expectation code for this example is "Performing routine procedures" (2.2.2).

- Question 3 (page 17): The topic codes for this example are (a) "Pythagorean Theorem and its applications" (1.3.3.2); and (b) "Rounding and significant figures" (1.1.5.2). Performance expectation codes for this example are (a) "Recalling mathematical objects and properties" (2.1.3); (b) "Performing routine procedures" (2.2.2); and (c) "Using more complex procedures" (2.2.3).

- Question 4 (page 17): The topic codes for this example are (a) "Angles" (1.3.2.2) and (b) "Parallelism and perpendicularity" (1.3.2.3). Performance expectation codes for this example are (a) "Relating representations" (2.5.2); (b) "Formulating and clarifying problems and situations" (2.3.1); and (c) "Solving" (2.3.3).

Create Chapter Summary Measures

After the chapter review questions were coded, the assigned curriculum topic and performance expectation codes were used to create summary measures for each textbook chapter. The chapter summary measures were the total content page count and a set of weighted topic content page counts. The Center for Research in Mathematics and Science Education at MSU generated the chapter summary measures for all textbook chapters coded for use in the MCS.

The total content page count represents the total amount of instructional material in the chapter. For each mathematics topic, the number of times that topic was coded in the chapter review questions was summed and then divided by the number of chapter review questions. This proportion was then multiplied by the number of pages in the textbook chapter, creating a value that represents the number of content pages devoted to that topic in the chapter. These values were summed across all mathematics topics to produce a total content page count for the chapter.

Both the amount of instructional material covered, which is measured by total subject matter content, and the tasks attempted, which is measured by performance expectations, contribute to how a student learns a subject. Therefore, both are key components in a quality curriculum (Gamoran et al. 1997). A weighted topic content page count represents the amount of instructional material within the chapter devoted to a mathematics topic, as weighted by ratings of the performance expectation codes assigned to the chapter review questions concerning the topic. The weight represents the topic's cognitive challenge. Cognitive challenge categorizes the complexity of the student tasks required to answer the chapter review questions, as measured by the performance expectation codes. Each performance expectation code assigned to a chapter review question was ranked on a four-point scale. A rank of 1 indicated the lowest level of complexity, such as recalling mathematical definitions or performing routine procedures. A rank of 4 indicated the highest level of complexity, such as problem solving or proving theorems. Chart A1 lists all of the performance expectation codes in Section 2. Chart A3 shows the four-point scale on which the performance expectations were ranked. If a chapter review question was assigned more than one performance expectation code, then the question's rank equaled the highest rank among the performance expectation codes.

Calculating the weighted topic content page counts were similar to calculating the total content page count. For each chapter review question, a coded topic received a value equal to the highest rank of performance expectations coded for that question. If the topic was not coded for the question, indicating it was absent, it received a value of zero. These values were summed for all chapter review questions and divided by the number of chapter review questions. This proportion was then multiplied by the number of pages in the textbook chapter to create a weighted topic content page count for each topic in the chapter.

To streamline the creation of the weighted topic content page counts, the over 200 mathematics topics from the TIMSS framework were aggregated into 32 general topic groupings commonly associated with elementary and secondary education mathematics textbooks. These 32 groupings reflected the basic hierarchical structure of the mathematics curriculum framework as vetted by mathematics curriculum experts worldwide (Robitaille et al. 1993). These groupings are an expansion upon the 29 groupings used for the AHAA study conducted in 2004 (Schiller et al. 2008). The "Numbers and Arithmetic" grouping was replaced by groupings for its three main subtopics, while the "Validation and Structuring" groupings was split into separate groups. These groupings covered all mathematics topics, not just algebra I and geometry. Chart A1 lists all of the over 200 mathematics topics in section 1, Curriculum Topics. Chart A2 shows how the framework aggregated the mathematics topics into the 32 general topic groupings (in the third column labeled "Initial grouping label").

For example, the mathematics topic illustrated by Question 1 from the "Understanding textbook coding" section (found on pages 16 and 17 of this report) was "Linear equations and their formal (closed) solutions" (mathematics framework code 1.6.2.5). For the chapter summaries, this topic was among the five topics collapsed into the "Basic equation" grouping. For Question 4, both the mathematics topics of "Angles" and "Parallelism and perpendicularity" (mathematics framework codes 1.3.2.2 and 1.3.2.3) were collapsed into the "Two-dimensional geometry" grouping.

Course Analyses

The second step of the study analyses focused on the courses. After the chapter review questions were coded and the information was summarized for chapters, course summary measures of the content of algebra I and geometry courses were created. A factor analysis was conducted to identify the patterns in the mathematics content coverage across courses. A discriminant analysis using the course summary measures and results of the factor analysis was conducted to classify the courses into four distinct content categories.

Course Summary Measures

The chapter summary measures were used to create the course summary measures, which were the total content page count and weighted topic content page counts. The chapters that a course covered formed the basis for calculating the course summary measures. Each school reported the textbook chapters that the instruction in each algebra I or geometry course was expected to cover. Over 400 different variations of the course chapters covered were identified across the approximately 2,000 algebra I and geometry courses in the study for which information for at least one coded textbook was available.

The total content page count for each textbook chapter that was covered in a course was summed to create the course's total content page count. For each of the 32 general topic groupings, the weighted topic content page count for each textbook chapter that was covered in the course was summed to produce the course's weighted topic content page count. A weighted percentage distribution of the 32 general topic groupings for the course was calculated from these weighted topic content page counts. The overall cognitive challenge rating for the course was calculated by summing the weighted topic content page counts across all topic groupings and dividing the sum by the total content page count.

An examination of the weighted percentage distributions of the approximately 2,000 algebra I and geometry courses with coded textbook data showed that the content of the algebra I and geometry courses varied widely. For example, among algebra I courses, the percentage of course content devoted to basic equations varied from 1 percent to 46 percent, with the average being just under 27 percent. While some topic groupings tended to cluster together, the relationships among topics were complex. For example, algebra I courses with a greater percentage of basic equations content also tended to have greater percentages of content in pre-equations and basic number theory, as well as smaller percentages in basic functions content. However, the percentage of content devoted to basic equations was not related to the percentages in advanced equations or advanced functions. The inclusion of different nonfocal topics (i.e., those topics other than algebra for algebra I courses and other than geometry for geometry courses) varied greatly across courses and averaged approximately one-third of subject matter content in both algebra I and geometry courses.

Factor Analysis: Identifying Patterns of Subject Matter Content Across Courses

The variation in the overall percentage distribution of the 32 general topic groupings and the clustering of some topic groupings revealed the complex relationships among the topics. Factor analysis was used to reduce the complex relationships among the mathematics topic groupings into a smaller number of algebra I and geometry course curriculum factors. A separate factor analysis was done for the two mathematics courses, and each analysis used the weighted percentage distribution of mathematics topic groupings. Weighted percentage distributions were used when conducting the factor analysis because they captured the relative content emphasis and no problems with the reliability of the factor scores were found. The factor analysis used principal component extraction with a varimax rotation procedure. The varimax rotation was used because it produced uncorrelated factors, which was necessary so that the factors generated from the factor analysis could be used in the subsequent discriminant analysis. Both the varimax orthogonal rotation and the direct oblimin oblique rotation methods were tested. While both approaches consistently identified the same number of correlated factors, the varimax orthogonal rotation method was used in the final factor analysis to address the multicollinearity among the topic groupings. The Kaiser criterion was used for factor selection, which determines the number of nontrivial latent dimensions in the input data by the number of eigenvalues from the input correlation matrix that are greater than 1.0. Factor analysis was used in this study solely for descriptive purposes—to mathematically summarize variation in topical content across courses. Both the Kaiser criterion and scree plots were selected as standard methods for determining the optimal number of factors that capture the vast majority of the underlying content variation. The Kaiser criterion and scree plots identified the same number of factors. The Kaiser criterion was used in the final analysis because it involved less subjectivity in determining the number of factors.

Initial factor analyses had problems of skewness due to the nonfocal mathematics topic groupings that rarely appear in algebra I and geometry textbooks. To address the skewness, the 32 general topic groupings used to calculate the chapter summary measures were aggregated to 17 topic groupings prior to the final factor analysis. Those topic groupings designated as algebra or geometry were not aggregated. Only nonfocal topic groupings (i.e., the groupings of mathematics topics that are not designated as algebra or geometry) were aggregated. There were two criteria required for aggregating the nonfocal topic groupings. First, topic groupings could be aggregated if they were taught at comparable grade levels internationally, as determined using the International Grade Placement (IGP) index from TIMSS (Schmidt et al. 1997). The IGP provides a composite among 40 international countries of at what grade levels mathematics topics are taught. What constituted "comparable" grade levels was in relation to high school algebra I and geometry courses. Topics normally taught before algebra I and geometry were considered comparable to one another, while topics normally taught after these courses were considered comparable to one another. For example, according to the IGP, both fractions and operations were primarily taught before high school, so they were considered taught at a comparable grade level, for the purposes of this analysis only. Second, topic groupings could be aggregated if the percentages of content in the categories were significantly positively related to each other, as determined by correlations across courses calculated in this study. For example, results from the MCS showed that courses with a greater percentage of content in fractions also tended to have a greater percentage of content in operations. Thus, the fractions topic grouping was aggregated with the operations topic grouping into a broader grouping called "Arithmetic." The goal was to have the combined topic groupings account for a sizeable amount of content without masking meaningful variation in topic coverage across courses. For example, if mathematics topics taught before algebra I and topics taught after geometry were combined, meaningful variations in topic coverage may not be evident. Chart A2 shows how the mathematics topic groupings were aggregated from 32 groupings (listed in the third column as "Initial grouping label") to 17 groupings (listed in the second column as "Factor analysis grouping label").

The five resulting algebra I factors constituted distinct patterns in mathematics course content. For example, the first algebra I factor focused on geometry content, as it was strongly associated with measures of vector and two- and three-dimensional geometry. The second factor concentrated less on introductory algebra, as it was negatively associated with measures of pre-equation and basic number theory. The third factor focused on higher level algebra, as it was associated with coverage of basic functions and data representation, but negatively associated with basic equations and coordinate geometry. The fourth factor concentrated more on higher level mathematics, as it was strongly associated with advanced mathematics and uncertainty and probability and negatively associated with arithmetic. The fifth factor was negatively associated with coverage of advanced equations and positively associated with advanced number theory. The five course curriculum factors accounted for 64 percent of the variation in content across algebra I courses. Table A1 lists the algebra I course factor analysis results.

The six resulting geometry factors also portrayed distinct patterns in mathematics course content. For example, the first factor indicated algebraic content within a geometry course. This factor was strongly associated with coverage of basic functions, advanced functions, advanced number theory, and data representation, and negatively associated with two-dimensional geometry. The second factor also focused on algebraic content, being strongly associated with the measures of basic number theory and advanced equations and negatively associated with pre-geometry. Factors 3, 4, and 5 had strong (but reversed) associations with at least one type of advanced geometry content, such as three-dimensional geometry or coordinate geometry. The sixth factor was strongly associated with advanced mathematics. The six factor-based indicators accounted for 77 percent of the variation in content across geometry courses. Table A1 lists the geometry course factor analysis results.

Discriminant Analysis: Classifying Courses Into Course Categories

When the analysis plan for this study was developed, there were no universally established criteria for what defined the curriculum or the rigor for algebra I and geometry courses. Therefore, information from the 2005 NAEP HSTS was used to develop empirically derived categorizations of algebra I and geometry courses based on patterns in subject matter content and degree of cognitive challenge in textbooks adopted by schools. The classification process was done separately for algebra I and geometry. Discriminant function analyses were conducted to create distinct course categories. The process behind the discriminant function analyses was to identify school courses by category (low, medium, high, or integrated), using the assumption that there were identifiable differences in curriculum content across the nation. In broad terms, low courses indicate mathematics courses that generally cover basic topics and offer simple or repetitive student exercises, while high courses indicate mathematics courses that generally include more advanced topics and pose more challenging exercises to the students. Medium courses resemble the content and challenge found in regular mathematics courses. The Classification of Secondary School Courses (CSSC) codes assigned to algebra I and geometry courses were used to develop criteria for classifying courses based on observed trends in curriculum content in different types of courses across the country. As part of the HSTS, each course was assigned a CSSC code by matching the course description from the high school course catalog to the course descriptions on the CSSC code list.

There were four algebra I courses distinguished by the CSSC: year one of two-year algebra I, year two of two-year algebra I, regular algebra I, and integrated (or unified) mathematics I. A two-year algebra I course is an algebra course designed to be taught in a two-year sequence. Year one reviews pre-algebra topics and teaches students to solve first-degree equations and inequalities, while the year two covers topics such as polynomial and quadratic equations with an emphasis on formal problem solving. A first-year integrated mathematics course interweaves algebra, geometry, trigonometry, analysis, statistics, and other mathematics topics into a single course that is generally taken at the same time most students take algebra I courses. There were also four geometry courses distinguished by the CSSC: informal geometry, regular geometry, honors geometry, and integrated (or unified) mathematics II. An informal geometry course is a simplified geometry course that focuses more on practical applications and less on proving theorems. An honors geometry course covers such topics as three-dimensional and coordinate geometry and incorporates formal proofs. A second-year integrated mathematics course interweaves algebra, geometry, trigonometry, analysis, statistics, and other mathematics topics into a single course that is generally taken at the same time most students take geometry courses.

A discriminant function analysis for algebra I courses generated predictions of classification into one of the four defined algebra I course categories. Classification of first-year integrated mathematics courses was based on how closely they matched a course with an integrated curriculum. Classification of the nonintegrated algebra I courses was based on how closely the course content mirrored either year one or year two of a two-year algebra I class or a regular algebra I class. The input data for this discriminant function analysis included the course summary measures of total content page count and overall cognitive challenge rating, as well as the five algebra I subject-matter content factors from the algebra I course factor analysis.

A discriminant function analysis for geometry courses generated predictions of classification into one of the four defined geometry course categories. Classification of second-year integrated mathematics courses was based on how closely the course content matched a course with an integrated curriculum. Classification of the nonintegrated geometry courses was based on how closely the course content mirrored either informal geometry, regular geometry, or honors geometry. The input data for this discriminant function analysis included the course summary measures of total content page count and overall cognitive challenge rating, as well as the six geometry subject-matter content factors from the geometry course factor analysis.

Courses were classified into course categories based on the probabilities calculated from the discriminant functions. If a course best matched an integrated curriculum, it was classified into the integrated category. Otherwise, the probability of a course being each CSSC classification was rank ordered and divided into thirds. These cutpoints in the range of probabilities were used to classify the courses into three categories—low, medium, and high. Considerations used to set the cutpoints and determine the number of course categories included: (a) that groups of courses were large enough that statistics generated for major student subgroups, such as student race/ethnicity, would meet the minimum reporting size (i.e., 62 or more observations); (b) that groups of courses were still relatively similar in subject matter content; and (c) that the interpretation of course types described in the study would apply for both algebra I and geometry. Initially, five groups of nonintegrated courses were considered in producing distinctive profiles in subject matter content, but it resulted in two of the groups being too small to meet statistical reporting standards. Based on the discriminant analysis results, approximately 67 percent of geometry courses and 71 percent of algebra I courses had predicted probabilities of approximately 0.4 (i.e., 40 percent) or greater of belonging to the CSSC code on which its course category was derived.

There were four different course categories that algebra I and geometry courses could be classified: low, medium, high, and integrated. The definitions of the course categories follow, including the minimum probabilities needed to be classified for each category. Table A3 shows the subject matter percentage breakdown for both integrated and nonintegrated algebra I and geometry courses.

Integrated Course Category: Courses that had a unified or integrated approach to mathematics were readily identified because they had extremely high similarities (e.g., probabilities greater than 0.7) to the distinctive characteristics of courses assigned an integrated course CSSC code. The probability of being an integrated course was also strongly negatively associated with the probability of being any of the other algebra I and geometry course types. An algebra I course that had a probability of 0.422 or higher of being an integrated course was assigned the integrated course category, while a geometry course that had a probability of 0.900 or higher of being an integrated course was assigned the integrated course category.

Low Course Category: A low course is defined as a mathematics course that covers the basic topics of the subject and/or has students gain knowledge of the subject through simple and repetitive exercises. For this study, a low course would most resemble year one of a two-year algebra I course or an informal geometry course. Low algebra I courses had higher probabilities of being associated with year one of a two-year algebra I course than being associated with year two of a two-year algebra I course. If an algebra I course had a probability of 0.489 or greater of being year one of a two-year algebra I course and a probability of 0.410 or less of being year two of a two-year course, then it was assigned the low algebra I course category. Low geometry courses had higher probabilities of being associated with an informal geometry course than being associated with an honors geometry course. If a geometry course had a probability of 0.158 or greater of being an informal geometry course, a probability of 0.208 or less of being an honors geometry course, and a higher probability of being an informal geometry course than an honors geometry course, then it was assigned the low geometry course category.

High Course Category: A high course is defined as a mathematics course that delves into advanced topics within the subject and/or challenges students' knowledge of the subject through exercises such as multistage problems and theorem proofs. For this study, a high course would most resemble year two of a two-year algebra I course or an honors geometry course. High algebra I courses had higher probabilities of being associated with year two of a two-year algebra I course than being associated with year one of a two-year algebra I course. If an algebra I course had a probability of 0.548 or greater of being year two of a two-year algebra I course and a probability of 0.321 or less of being year one of a two-year algebra I course, then it was assigned the high algebra I course category. High geometry courses had higher probabilities of being associated with an honors geometry course than being associated with an informal geometry course. If a geometry course had a probability of 0.294 or greater of being an honors geometry course and a probability of 0.059 or less of being an informal geometry course, then it was assigned a high geometry course category.

Medium Course Category: A medium course is defined as a course that covers both basic and advanced topics, and the exercises used to measure students' knowledge of the subject ranges from easy to challenging. For this study, a medium course would most resemble a regular algebra I or geometry course. Algebra I and geometry courses not classified as integrated, low, or high courses were assigned to the medium course category.

Student Analyses

The third step of the study analyses focused on the students who took algebra I and geometry courses. In this final step, summaries of the data for reporting purposes were developed that described the students who completed the courses. To develop the summaries, the course summaries were linked to the students, patterns in topic coverage were identified through factor analysis, and discriminant analysis was used to classify students' coursework into course levels.

Student Summary Measures

High schools frequently offer multiple algebra I and geometry courses, and students often take more than one such course. Approximately 50 percent of the MCS schools identified more than one algebra I course in their catalogs or transcripts, while 56 percent had more than one geometry course. The course summary measures were matched to students through the algebra I and geometry courses on students' transcripts. Because of the different ways schools record students' coursetaking on their transcripts, a large number of high school students in this study had multiple entries for algebra I or geometry on their high school transcripts. All cases were examined to determine whether the multiple entries represented a single or multiple courses.

Creating the student summary measures involved processing the course summary measures (i.e., the total content page count, the overall cognitive challenge rating, and the weighted content page counts). How the student summary measurements were created depended on the number of algebra I or geometry course entries recorded on the student's transcript. If there was a single entry on the transcript, or there were multiple entries for the same course (e.g., when students had separate transcript records for the first and second semesters of the same algebra I course), then the student summary measures were equal to the course summary measures. If there were multiple entries on the transcript that represented different courses (e.g., a student repeated an algebra I course that used a different textbook, or a student switched midyear from a regular geometry course to an honors geometry course), then each set of course summary measures was weighted by the course credits earned for the course and then standardized to a year-long (i.e., one credit) course.

In situations where there were multiple entries on the transcript that represented different courses, the total content page count and the weighted content page counts for each course were weighted by the course credits the student earned for the course. If a student did not pass the course and earned no course credits, then it was weighted by the course credits the student would have earned for passing the course. These counts were summed across all courses and then divided by the total number of course credits earned by the student. These calculations resulted in the student's total content page count and weighted content page counts. These weighted content page counts were then summed across all topics, and the sum was used to calculate a weighted percentage distribution of the mathematics topic groupings for the student. The student's overall cognitive challenge rating was also calculated by dividing the sum of its weighted topic content page counts by its total content page count.

Factor Analysis: Identifying Patterns of Subject Matter Content Across Students

After the student summary measures were created, the information was used in factor analyses to identify the patterns of the mathematics topics across students who took algebra I and geometry courses. Separate factor analyses were done for the two mathematics courses, and each analysis used the weighted percentage distribution of mathematics topic groupings calculated for students. Factor analyses were run for students who took the courses, using the same procedures as were done for the course factor analyses (i.e., a principal component analysis with a varimax rotation procedure, and using the Kaiser criterion for factor selection). The uncorrelated factors produced were then used in the discriminant function analyses.

The 17 mathematics topic groupings that were aggregated during the course analyses from the 32 general mathematics topic groupings were also used in this factor analysis. The results from the factor analyses showed similar patterns of subject matter groupings across students as were found across courses. For students who took one or more algebra I courses, factors 2 and 3 corresponded to factors 3 and 2, respectively, from the algebra I course factor analysis. The other three factors showed quite similar loadings. The five underlying factors of cumulative algebra I course content accounted for 67 percent of the variation in the 17 mathematics topic groupings. For students who took one or more geometry courses, factors 1, 4, and 5 had similar factor loadings to factors 1, 5, and 4, respectively, from the geometry course factor analysis. Factors 2, 3 and 6 showed different patterns from the course factor analyses. The six underlying factors of cumulative geometry content indicators accounted for nearly 80 percent of the variation in the 17 mathematics topic groupings. Table A2 lists the student factor analysis results.

Discriminant Analysis: Classifying Student Coursework Into Course Levels

The purpose of the discriminant function analyses was to create distinct categories of students' algebra I and geometry coursework.

These categories of student coursework are identified as "course levels" within this report. The classification process was done separately for algebra I and geometry coursework. For these discriminant function analyses, students' coursework was not characterized using CSSC codes, because a given student could enroll in more than one course with different codes (e.g., first and second year of algebra I or honors and regular geometry). The algebra I or geometry curricula taken by students throughout high school were instead classified based on the previously defined low, medium, high, or integrated course categories, which served as the dependent variable for the student discriminant analyses. The input data for the discriminant function analyses included the student summary measures of the total content page count and the overall cognitive challenge ranking, along with the subject-matter content indicators from the student factor analysis (five factors for algebra I and six factors for geometry).

The estimated discriminant functions generated predictions of classification into the three course categories in each subject. Student coursework could also be categorized as integrated courses. For cases in which the course categories defined by the course discriminant analysis could be compared to the student discriminant analysis results, approximately 95 percent of students' geometry coursework and 91 percent of students' algebra I coursework had probabilities of approximately 0.4 or greater of being classified as their given course category. The final course levels were based on the highest classification probability generated from the student discriminant analysis. Students who had enrolled in courses with different course categorizations (e.g., both low and high because they had enrolled in both years of two-year algebra I) were then classified into the course level that most closely matched their overall curriculum profile (i.e., had the highest probability).

If a student took different types of algebra I courses, such as a regular algebra I course and honors algebra I course, then the student was classified as having mixed algebra I course types and was not included in the results presented in the Comparison of School Courses section of the report. Students who took different types of geometry courses were treated in the same manner. Approximately 3.0 percent of the students in the algebra I sample had mixed algebra I course types, while approximately 2.7 percent of the students in the geometry sample had mixed geometry course types.

Course Levels

Students' algebra I and geometry coursework could be classified into one of three course levels—beginner, intermediate, and rigorous—or as an integrated mathematics course. Each student was assigned a course level based on the highest probability, as generated by the student discriminant function analysis, that the student's coursework could be categorized as a beginner, intermediate, rigorous, or integrated course. The beginner, intermediate, and rigorous course levels are analogous to the low, medium, and high course categories, respectively. The difference between the two sets of measures is the level of the analysis. The course categories are generated at the course level and measure the content and challenge of a single algebra I or geometry course that was listed on the student's transcript, while the course levels are generated at the student level and measure the overall content and challenge of a student's coursework in all algebra I or geometry courses that were listed on the student's transcript. The definitions of the course levels follow. Table A4 shows the subject matter percentage breakdown for both integrated and nonintegrated algebra I and geometry coursework among students.

Integrated Mathematics Courses: If a student's coursework most associated with an integrated or unified approach to mathematics, then the student was classified as having taken an integrated mathematics course. The probability of being an integrated mathematics course must be higher than the probabilities of being any of the other algebra I and geometry course levels. As noted in the report, integrated courses were not ranked by course level due to the small number of high school graduates who took integrated mathematics courses.

Beginner Level Courses: If a student's coursework most associated with the low course category, then the student was classified as having taken a beginner level course. The probability of being a beginner level course must be higher than the probabilities of being intermediate level, rigorous level, or integrated mathematics courses. A beginner level algebra I course most associated with year one of a two-year algebra I course, while a beginner level geometry course most associated with an informal geometry course.

Intermediate Level Courses: If a student's coursework most associated with the medium course category, then the student was classified as having taken an intermediate level course. The probability of being an intermediate level course must be higher than the probabilities of being beginner level, rigorous level, or integrated mathematics courses. An intermediate level algebra I course most associated with a regular algebra I course, while an intermediate level geometry course most associated with a regular geometry course.

Rigorous Level Courses: If a student's coursework most associated with the high course category, then the student was classified as having taken a rigorous level course. The probability of being a rigorous level course must be higher than the probabilities of being beginner level, intermediate level, or integrated mathematics courses. A rigorous level algebra I course most associated with year two of a two-year algebra I course, while a rigorous level geometry course most associated with an honors geometry course

The labels that were assigned to the three course level measures and the integrated course category describe the overall difficulty of each level, as defined by curriculum content and cognitive challenge. The beginner, intermediate, and rigorous labels reflect the hierarchical nature of the course levels, from the least amount of difficulty to the most amount of difficulty. The labels were also deliberately chosen because they contrast with the names that schools assigned their mathematics courses. While the rigorous course level is the highest level, it was not labeled "advanced" as to differentiate the level from the higher level courses that schools often label "advanced" (e.g., Advanced Algebra/ Algebra III and Advanced Geometry). These advanced courses include advanced mathematics topics that are generally not covered in algebra I and geometry courses.

Curriculum Topics

The percentage distribution of students' algebra I and geometry courses by curriculum topics were reported in the Mathematics Course Profiles section of the report. The 17 mathematics topic groupings used in the course and student factor analyses provided too much detail to concisely characterize the differences in students' algebra I and geometry courses for the report. Therefore, the 17 mathematics topic groupings were collapsed to six broad categories. Both the algebra and geometry topic groupings were aggregated into introductory and advanced categories. The remaining mathematics topic groupings were assigned to categories that indicated whether the topics were traditionally taught before an algebra I course or after a geometry course. The six resulting mathematics topic categories are identified as the "curriculum topic categories" in table A of the Executive Summary and "curriculum topic groups" in figures 1, 3, and 5 and tables 2, 3, and 4 in the Mathematics Course Profiles section. Chart A2 shows how the 17 mathematics topic groupings (in the second column marked "Factor analysis grouping label") were collapsed into the six mathematics topic categories used for reporting (in the first column marked "Main curriculum category").

The definitions of the six main curriculum topic categories are as follows:

Elementary and Middle School Mathematics: This category includes mathematics topics that are traditionally taught before a student takes an algebra I course. These topics include elements of basic arithmetic (e.g., addition, subtraction, fractions, and rounding) and pre-geometry (e.g., patterns, perimeter, area, and proportion).

Introductory Algebra: This category includes mathematics topics needed to understand the basics of algebra and provide the foundation for learning advanced algebra. These topics include pre-algebra, basic algebraic equations (e.g., algebraic expression, simple linear equations, and simple inequalities), and the basic elements of number theory (e.g., integers, absolute value, and rational numbers).

Advanced Algebra: This category includes mathematics topics that cover the more complex elements of algebra. These topics include advanced equations (e.g., quadratic equations, polynomial equations, and matrix solutions), basic functions (e.g., representation of relationships and functions, and graphing functions), advanced functions (e.g., functions of several variables and quadratic functions), and advanced number theory (e.g., real numbers, exponents, roots, radicals, and matrices).

Two-Dimensional Geometry: This category includes mathematics topics that focus on basic linear and planar geometric concepts. Examples of topics in this category include basic geometric concepts (e.g., points, angles, parallelism, and perpendicularity) and the properties of shapes.

Advanced Geometry: This category includes mathematics topics that cover advanced geometric concepts such as three-dimensional geometry (e.g., three-dimensional shapes, conic sections), coordinate geometry (e.g., equations of lines, planes, and surfaces in space), and vector geometry (e.g., vectors, transformation, congruence, and similarity).

Other High School Mathematics: This category includes mathematics topics that are traditionally taught in courses taken after geometry and algebra II. Examples of topics in this category include trigonometry, pre-calculus, statistics (e.g., data representation and analysis, uncertainty and probability), validation and structuring (e.g., logic, set theory, and axioms), discrete mathematics (e.g., tree diagrams and binary arithmetic), finite mathematics, and calculus.

Weighting and Variance Estimation

In the same way that schools and students participating in the HSTS were chosen to be nationally representative of public and private high school graduates, the schools and students participating in the MCS were selected to be representative of those same graduates. The results from the NAEP twelfth-grade mathematics assessment were included to provide accurate estimates of overall student performance. Results are weighted to take into account the fact that schools and students represent different proportions of the overall populations. All estimates were weighted to provide unbiased estimates of the national population.

The school weights for the 2005 NAEP HSTS participating schools served as the basis for applying textbook nonresponse adjustments to compensate for textbook nonresponse. The 2005 NAEP HSTS sampling weights for schools and students in the textbook study were adjusted to compensate for the loss of 2005 NAEP HSTS participating schools that offered algebra I and geometry classes but did not provide textbook data for this study. The technical details of the original 2005 NAEP HSTS sampling weights are described in The 2005 High School Transcript Study User's Guide and Technical Report (Shettle et al. 2008).

The school weights of the nonresponding schools were distributed to those responding schools within weighting classes. The weighting classes were defined following the classification criteria adopted for the 2005 NAEP HSTS. The adjustment factor for each class was prorated by total student enrollment among the responding and nonresponding schools. Both linked and unlinked samples were weighted to represent the national population. Two sets of adjustment factors were defined—for all 2005 NAEP HSTS participating schools (unlinked weights) and for the 2005 NAEP HSTS schools that also participated in 2005 NAEP (linked weights). The school weights were used in analyzing the variation in curriculum content across courses during the development of course-level indicators to adjust for differences in probabilities that schools were selected for and participated in the 2005 NAEP HSTS.

The student weights were computed using the textbook-compensated school weights; that is, the school weights that were revised for textbook nonresponse. The sequence of steps for student weighting was the same as those used for the 2005 NAEP HSTS students. For the student base weights, the textbook-compensated school weights were used to replace the school nonresponse adjusted weights. All adjustments for nonresponse and weight trimming were conducted in the same manner, again producing two sets of student weights—for students in all 2005 NAEP HSTS participating schools (unlinked weights) and the students in 2005 NAEP HSTS that were also 2005 NAEP participating schools (linked weights). The student weights were used to adjust for differences in sampling probabilities of the 12,500 students who had taken algebra I and the 15,900 students who had taken geometry during high school. All results generated for the MCS were at the student-level and incorporated either the unlinked or linked weights. The replicate weights for variance estimation were calculated along with the full sample weights, repeating each adjustment in the same manner as the full sample weights.

High school student estimates for the MCS were subject to sampling error because they were derived from a sample, rather than the whole population. Sampling error was measured by the sampling variance, which indicated how much the population estimate for a given statistic was likely to change if it had been based on another equivalent sample of individuals drawn in exactly the same manner as the actual sample. Variances were estimated using jackknife replication methods (Krewski and Rao 1981). This estimation involved measuring the variability among subsamples (replicates) to generate an accurate estimate of variance for the full sample.

Interpreting Statistical Significance

Comparisons over time or between groups are based on statistical tests that consider both the estimated size of the difference and the standard error of that estimated difference. When a difference—such as the difference between the average scores of two groups—has a large standard error, a numerical difference that seems large may not be statistically significant (i.e., a null hypothesis of no difference cannot be rejected with sufficient confidence). Differences of the same estimated size may be statistically significant in some cases but not others, depending on the sizes of the standard errors involved. For this report, only those differences that are found to be statistically significant are discussed as higher or lower. In conducting the statistical significance tests used in this report, no adjustments were made for multiple comparisons.

CHART A1. Mathematics framework curriculum topics and performance expectations

1. Curriculum Topics
   - 1.1. Numbers
     - 1.1.1. Whole numbers
       - 1.1.1.1. Meaning
         - 1.1.1.1.1. The uses of numbers
         - 1.1.1.1.2. Place value and numeration
         - 1.1.1.1.3. Ordering and comparing numbers
       - 1.1.1.2. Operations
         - 1.1.1.2.1. Addition
         - 1.1.1.2.2. Subtraction
         - 1.1.1.2.3. Multiplication
         - 1.1.1.2.4. Division
         - 1.1.1.2.5. Mixed operations
       - 1.1.1.3. Properties of operations
         - 1.1.1.3.1. Associative properties
         - 1.1.1.3.2. Commutative properties
         - 1.1.1.3.3. Identity properties
         - 1.1.1.3.4. Distributive properties
         - 1.1.1.3.5. Other number properties
     - 1.1.2. Fractions and decimals
       - 1.1.2.1. Common fractions
         - 1.1.2.1.1. Meaning and representation of common fractions
         - 1.1.2.1.2. Computations with common fractions and mixed numbers
       - 1.1.2.2. Decimal fractions
         - 1.1.2.2.1. Meaning and representation of decimals
         - 1.1.2.2.2. Computations with decimals
       - 1.1.2.3. Relationships of common and decimal fractions
         - 1.1.2.3.1. Conversion to equivalent forms
         - 1.1.2.3.2. Ordering of fractions and decimals
       - 1.1.2.4. Percentages
         - 1.1.2.4.1. Percent computations
         - 1.1.2.4.2. Various types of percent problems
       - 1.1.2.5. Properties of common and decimal fractions
         - 1.1.2.5.1. Associative properties
         - 1.1.2.5.2. Commutative properties
         - 1.1.2.5.3. Identity properties
         - 1.1.2.5.4. Inverse properties
         - 1.1.2.5.5. Distributive properties
         - 1.1.2.5.6. Cancellation properties
         - 1.1.2.5.7. Other number properties
     - 1.1.3. Integers, rational and real numbers
       - 1.1.3.1. Negative numbers, integers, and their properties
         - 1.1.3.1.1. Concept of integers
         - 1.1.3.1.2. Operations with integers
         - 1.1.3.1.3. Concept of absolute value
         - 1.1.3.1.4. Properties of integers

CHART A1. Mathematics framework curriculum topics and performance expectations (continued)

- 1.1.3.2. Rational numbers and their properties
  - 1.1.3.2.1. Concept of rational numbers
  - 1.1.3.2.2. Operations with rational numbers
  - 1.1.3.2.3. Properties of rational numbers
  - 1.1.3.2.4. Equivalence of differing forms of rational numbers
  - 1.1.3.2.5. Relation of rational numbers to terminating and recurring decimals
- 1.1.3.3. Real numbers, their subsets, and properties
  - 1.1.3.3.1. Concept of real numbers (including concept of irrationals)
  - 1.1.3.3.2. Subsets of real numbers (integers, rational numbers, etc.)
  - 1.1.3.3.3. Operations with real numbers and absolute value
  - 1.1.3.3.4. Properties of real numbers (including density, order, properties of absolute value, completeness, etc.)

1.1.4. Other numbers and number concepts
- 1.1.4.1. Binary arithmetic and other number bases
- 1.1.4.2. Exponents, roots, and radicals
  - 1.1.4.2.1. Integer exponents and their properties
  - 1.1.4.2.2. Rational exponents and their properties
  - 1.1.4.2.3. Roots and radicals and their relation to rational exponents
  - 1.1.4.2.4. Real exponents
- 1.1.4.3. Complex numbers and their properties
  - 1.1.4.3.1. Concept of complex numbers
  - 1.1.4.3.2. Algebraic form of complex numbers and their properties
  - 1.1.4.3.3. Trigonometric form of complex numbers and their properties
  - 1.1.4.3.4. Relation of algebraic and trigonometric form of complex numbers
- 1.1.4.4. Number theory
  - 1.1.4.4.1. Primes and factorization
  - 1.1.4.4.2. Elementary number theory,
- 1.1.4.5. Systematic counting
  - 1.1.4.5.1. Tree diagrams and other forms of systematic counting
  - 1.1.4.5.2. Permutations, combinations
- 1.1.4.6. Matrices
  - 1.1.4.6.1. Concept of a matrix
  - 1.1.4.6.2. Operations with matrices
  - 1.1.4.6.3. Properties of matrices

1.1.5. Estimation and number sense
- 1.1.5.1. Estimating quantity and size
- 1.1.5.2. Rounding and significant figures
- 1.1.5.3. Estimating computations
  - 1.1.5.3.1. Mental arithmetic
  - 1.1.5.3.2. Reasonableness of results
- 1.1.5.4. Exponents and orders of magnitude

1.2. Measurement

1.2.1. Units
- 1.2.1.1. Concept of measure (including nonstandard units)
- 1.2.1.2. Standard units (including metric system)
- 1.2.1.3. Use of appropriate instruments
- 1.2.1.4. Common measures (length, area, volume, time, calendar, money, temperature, mass, weight, angles)
- 1.2.1.5. Quotients and products of units (km/h, m/s, etc.)
- 1.2.1.6. Dimensional analysis

CHART A1. Mathematics framework curriculum topics and performance expectations (continued)

- 1.2.2. Computations and properties of length, perimeter, area, and volume
  - 1.2.2.1. Computations, formulas and properties of length and perimeter
  - 1.2.2.2. Computations, formulas and properties of area
  - 1.2.2.3. Computations, formulas and properties of surface area
  - 1.2.2.4. Computations, formulas and properties of volumes
- 1.2.3. Estimation and error
  - 1.2.3.1. Estimation of measurement and errors of measurement
  - 1.2.3.2. Precision and accuracy of measurement

1.3. Geometry: position, visualization, and shape

- 1.3.1. One- and two-dimensional coordinate geometry
  - 1.3.1.1. Line and coordinate graphs
  - 1.3.1.2. Equations of lines in a plane
  - 1.3.1.3. Conic sections and their equations
- 1.3.2. Two-dimensional geometry basics
  - 1.3.2.1. Points, lines, segments, half-lines, and rays
  - 1.3.2.2. Angles
  - 1.3.2.3. Parallelism and perpendicularity
- 1.3.3. Polygons and circles
  - 1.3.3.1. Triangles and quadrilaterals: their classification and properties
  - 1.3.3.2. Pythagorean Theorem and its applications
  - 1.3.3.3. Other polygons and their properties
  - 1.3.3.4. Circles and their properties
- 1.3.4. Three-dimensional geometry
  - 1.3.4.1. Three-dimensional shapes and surfaces and their properties
  - 1.3.4.2. Planes and lines in space
  - 1.3.4.3. Spatial perception and visualization
  - 1.3.4.4. Coordinate systems in three dimensions
  - 1.3.4.5. Equations of lines, planes and surfaces in space
- 1.3.5. Vectors
- 1.3.6. Simple topology

1.4. Geometry: symmetry, congruence, and similarity

- 1.4.1. Transformations
  - 1.4.1.1. Patterns, tessellations, friezes, stencils
  - 1.4.1.2. Symmetry
  - 1.4.1.3. Transformations
- 1.4.2. Congruence and similarity
  - 1.4.2.1. Congruence
  - 1.4.2.2. Similarities (similar triangles and their properties, other similar figures and properties)
- 1.4.3. Constructions with straight-edge and compass

1.5. Proportionality

- 1.5.1. Proportionality concepts
  - 1.5.1.1. Meaning of ratio and proportion
  - 1.5.1.2. Direct and inverse proportion
- 1.5.2. Proportionality problems
  - 1.5.2.1. Solving proportional equations
  - 1.5.2.2. Solving practical problems with proportionality
  - 1.5.2.3. Scales (maps and plans)
  - 1.5.2.4. Proportion based on similarity

CHART A1. Mathematics framework curriculum topics and performance expectations (continued)

1.5.3. Slope and simple trigonometry
 1.5.3.1. Slope and gradient in straight line graphs
 1.5.3.2. Trigonometry of right triangles
1.5.4. Linear interpolation and extrapolation
1.6. Functions, relations, and equations
1.6.1. Patterns, relations, and functions
 1.6.1.1. Number patterns
 1.6.1.2. Relations and their properties
 1.6.1.3. Functions and their properties
 1.6.1.4. Representation of relations and functions
 1.6.1.5. Families of functions (graphs and properties)
 1.6.1.6. Operations on functions
 1.6.1.7. Related functions (inverse, derivative, etc.)
 1.6.1.8. Relationship of functions and equations (e.g., zeroes of functions as roots of equations)
 1.6.1.9. Interpretation of function graphs
 1.6.1.10. Functions of several variables
 1.6.1.11. Recursion
 1.6.1.12. Linear functions
 1.6.1.13. Quadratic functions
 1.6.1.14. Logarithmic and exponential functions
 1.6.1.15. Trigonometric functions
1.6.2. Equations and formulas
 1.6.2.1. Representation of numerical situations by equations
 1.6.2.2. Informal solution of simple equations
 1.6.2.3. Operations with expressions and evaluating expressions
 1.6.2.4. Equivalent expressions (including factorization and simplification)
 1.6.2.5. Linear equations and their formal (closed) solutions
 1.6.2.6. Quadratic equations and their formal (closed) solutions
 1.6.2.7. Polynomial equations and their solutions
 1.6.2.8. Trigonometrical equations and identities
 1.6.2.9. Logarithmic and exponential equations and their solutions
 1.6.2.10. Solution of equations reducing to quadratics, radical equations, absolute value equations, etc.
 1.6.2.11. Other solution methods for equations (e.g., successive approximation)
 1.6.2.12. Inequalities and their graphical representation
 1.6.2.13. Systems of equations and their solutions (including matrix solutions)
 1.6.2.14. Systems of inequalities
 1.6.2.15. Substituting into or rearranging formulas
 1.6.2.16. General equation of the second degree and its interpretation
1.6.3. Trigonometry and analytic geometry
 1.6.3.1. Angle measures: radians and degrees
 1.6.3.2. Law of sines and cosines
 1.6.3.3. Unit circle and trigonometric functions
 1.6.3.4. Parametric equations
 1.6.3.5. Polar coordinates
 1.6.3.6. Polar equations and their graphs

CHART A1. Mathematics framework curriculum topics and performance expectations (continued)

- 1.7. Data representation, probability, and statistics
  - 1.7.1. Data representation and analysis
    - 1.7.1.1. Collecting data from experiments and simple surveys
    - 1.7.1.2. Representing data
    - 1.7.1.3. Interpreting tables, charts, plots and graphs
    - 1.7.1.4. Kinds of scales (nominal, ordinal, interval, ratio)
    - 1.7.1.5. Measures of central tendency
    - 1.7.1.6. Measures of dispersion
    - 1.7.1.7. Sampling, randomness, and bias related to data samples
    - 1.7.1.8. Prediction and inferences from data
    - 1.7.1.9. Fitting lines and curves to data
    - 1.7.1.10. Correlations and other measures of relations
    - 1.7.1.11. Use and misuse of statistics
  - 1.7.2. Uncertainty and probability
    - 1.7.2.1. Informal likelihoods and the vocabulary of likelihoods
    - 1.7.2.2. Numerical probability and probability models
    - 1.7.2.3. Counting principles
    - 1.7.2.4. Mutually exclusive events
    - 1.7.2.5. Conditional probability and independent events
    - 1.7.2.6. Bayes' Theorem
    - 1.7.2.7. Contingency tables
    - 1.7.2.8. Probability distributions for discrete random variables
    - 1.7.2.9. Probability distributions for continuous random variables
    - 1.7.2.10. Expectation and the algebra of expectations
    - 1.7.2.11. Sampling (distributions and populations)
    - 1.7.2.12. Estimation of population parameters
    - 1.7.2.13. Hypothesis testing
    - 1.7.2.14. Confidence intervals
    - 1.7.2.15. Bivariate distributions
    - 1.7.2.16. Markov processes
    - 1.7.2.17. Monte Carlo methods and computer simulations
- 1.8. Elementary analysis
  - 1.8.1. Infinite processes
    - 1.8.1.1. Arithmetic and geometric sequences
    - 1.8.1.2. Arithmetic and geometric series
    - 1.8.1.3. Binomial Theorem
    - 1.8.1.4. Other sequences and series
    - 1.8.1.5. Limits and convergence of series
    - 1.8.1.6. Limits and convergence of functions
    - 1.8.1.7. Continuity
  - 1.8.2. Change
    - 1.8.2.1. Growth and decay
    - 1.8.2.2. Differentiation
    - 1.8.2.3. Integration
    - 1.8.2.4. Differential equations
    - 1.8.2.5. Partial differentiation

CHART A1. Mathematics framework curriculum topics and performance expectations (continued)

- 1.9. Validation and structure
  - 1.9.1. Validation and justification
    - 1.9.1.1. Logical connectives
    - 1.9.1.2. Quantifiers ("for all," "there exists")
    - 1.9.1.3. Boolean algebra and truth tables
    - 1.9.1.4. Conditional statements, equivalence of statements (including converse, contrapositive, and inverse)
    - 1.9.1.5. Inference schemes (e.g., modus ponens, modus tollens)
    - 1.9.1.6. Direct deductive proofs
    - 1.9.1.7. Indirect proofs and proof by contradiction
    - 1.9.1.8. Proof by mathematical induction
    - 1.9.1.9. Consistency and independence of axiom systems
  - 1.9.2. Structuring and abstracting
    - 1.9.2.1. Sets, set notation and set combinations
    - 1.9.2.2. Equivalence relations, partitions and classes
    - 1.9.2.3. Groups
    - 1.9.2.4. Fields
    - 1.9.2.5. Linear (vector) spaces
    - 1.9.2.6. Subgroups, subspaces, etc.
    - 1.9.2.7. Other axiomatic systems
    - 1.9.2.8. Isomorphism
    - 1.9.2.9. Homomorphism
- 1.10. Other content
  - 1.10.1. Informatics (operation of computers, flow charts, learning a programming language, programs, algorithms with applications to the computer, complexity)
  - 1.10.2. History and nature of mathematics
  - 1.10.3. Special applications of mathematics (kinematics, Newtonian mechanics, population growth, networks, linear programming, critical path analysis, economics examples)
  - 1.10.4. Problem solving heuristics
  - 1.10.5. Nonmathematical science content
  - 1.10.6 Nonmathematical content other than science

2. Performance Expectations
- 2.1. Knowing
  - 2.1.1. Representing
  - 2.1.2. Reorganizing equivalents
  - 2.1.3. Recalling mathematical objects and properties
- 2.2. Using routine procedures
  - 2.2.1. Using equipment
  - 2.2.2. Performing routine procedures
  - 2.2.3. Using more complex procedures
- 2.3. Investigating and problem solving
  - 2.3.1. Formulating and clarifying problems and situations
  - 2.3.2. Developing strategy
  - 2.3.3. Solving
  - 2.3.4. Predicting
  - 2.3.5. Verifying

CHART A1. Mathematics framework curriculum topics and performance expectations (continued)

- 2.4. Mathematical reasoning
    - 2.4.1. Developing notion and vocabulary
    - 2.4.2. Developing algorithms
    - 2.4.3. Generalizing
    - 2.4.4. Conjecturing
    - 2.4.5. Justifying and proving
    - 2.4.6. Axiomatizing
- 2.5. Communicating
    - 2.5.1. Using vocabulary and notation
    - 2.5.2. Relating representations
    - 2.5.3. Describing/discussing
    - 2.5.4. Critiquing

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

CHART A2. Aggregation of the Trends in International Mathematics and Science Study (TIMSS) mathematics curriculum framework topics to produce the six main curriculum categories for the Mathematics Curriculum Study: 2005

Main curriculum category	Factor analysis grouping label	Initial grouping label	Original framework codes
Elementary and middle school mathematics	Arithmetic	Meaning	1.1.1.1.1 1.1.1.1.2 1.1.1.1.3
		Operations	1.1.1.2.1 1.1.1.2.2 1.1.1.2.3 1.1.1.2.4 1.1.1.2.5
		Properties of operations	1.1.1.3.1 1.1.1.3.2 1.1.1.3.3 1.1.1.3.4 1.1.1.3.5
		Fractions	1.1.2.1.1 1.1.2.1.2 1.1.2.2.1 1.1.2.2.2 1.1.2.3.1 1.1.2.3.2 1.1.2.4.1 1.1.2.4.2 1.1.2.5.1 1.1.2.5.2 1.1.2.5.3 1.1.2.5.4 1.1.2.5.5 1.1.2.5.6 1.1.2.5.7
		Number theory	1.1.4.4.1 1.1.4.4.2
		Estimation	1.1.5.1 1.1.5.2 1.1.5.3.1 1.1.5.3.2 1.1.5.4
		Measurement	1.2.1.1 1.2.1.2 1.2.1.3 1.2.1.4 1.2.1.5 1.2.1.6 1.2.3.1 1.2.3.2
		Proportionality concepts	1.5.1.1 1.5.1.2
	Pre-geometry	Patterns	1.6.1.1
		Perimeter, area, volume	1.2.2.1 1.2.2.2 1.2.2.3 1.2.2.4
		Proportionality problems	1.5.2.1 1.5.2.2 1.5.2.3 1.5.2.4
Introductory algebra	Pre-equation	Pre-equation	1.6.2.1 1.6.2.2
	Basic number theory	Basic number theory	1.1.3.1.1 1.1.3.1.2 1.1.3.1.3 1.1.3.1.4 1.1.3.2.1 1.1.3.2.2 1.1.3.2.3 1.1.3.2.4 1.1.3.2.5
	Basic equations	Basic equations	1.6.2.3 1.6.2.4 1.6.2.5 1.6.2.12 1.6.2.15
Advanced algebra	Advanced equations	Advanced equations	1.6.2.6 1.6.2.7 1.6.2.8 1.6.2.9 1.6.2.10 1.6.2.11 1.6.2.13 1.6.2.14 1.6.2.16
	Basic functions	Basic functions	1.6.1.2 1.6.1.3 1.6.1.4 1.6.1.5 1.6.1.9 1.6.1.12
	Advanced functions	Advanced functions	1.6.1.6 1.6.1.7 1.6.1.8 1.6.1.10 1.6.1.11 1.6.1.13 1.6.1.14 1.6.1.15
	Advanced number theory	Advanced number theory	1.1.3.3.1 1.1.3.3.2 1.1.3.3.3 1.1.3.3.4 1.1.4.2.1 1.1.4.2.2 1.1.4.2.3 1.1.4.2.4 1.1.4.3.1 1.1.4.3.2 1.1.4.3.3 1.1.4.3.4 1.1.4.6.1 1.1.4.6.2 1.1.4.6.3

CHART A2. Aggregation of the Trends in International Mathematics and Science Study (TIMSS) mathematics curriculum framework topics to produce the six main curriculum categories for the Mathematics Curriculum Study: 2005 (continued)

Main curriculum category	Factor analysis grouping label	Initial grouping label	Original framework codes
Two-dimensional geometry	Two-dimensional geometry	Two-dimensional geometry	1.3.2.1 1.3.2.2 1.3.2.3 1.3.3.1 1.3.3.2 1.3.3.3 1.3.3.4
Advanced geometry	Three-dimensional geometry	Three-dimensional geometry	1.3.4.1 1.3.4.2 1.3.4.3 1.3.4.4 1.3.4.5
	Coordinate geometry	Coordinate geometry	1.3.1.1 1.3.1.2 1.3.1.3 1.5.3.1
	Vectors, transformation, congruence and similarity	Vectors, transformation, congruence and similarity	1.3.5 1.3.6 1.4.1.1 1.4.1.2 1.4.1.3 1.4.2.1 1.4.2.2 1.4.3
Other high school mathematics	Data representation and analysis	Data representation and analysis	1.7.1.1 1.7.1.2 1.7.1.3 1.7.1.4 1.7.1.5 1.7.1.6 1.7.1.7 1.7.1.8 1.7.1.9 1.7.1.10 1.7.1.11
	Uncertainty and probability	Uncertainty and probability	1.7.2.1 1.7.2.2 1.7.2.3 1.7.2.4 1.7.2.5 1.7.2.6 1.7.2.7 1.7.2.8 1.7.2.9 1.7.2.10 1.7.2.11 1.7.2.12 1.7.2.13 1.7.2.14 1.7.2.15 1.7.2.16 1.7.2.17
	Other high school topics	Discrete math	1.1.4.1 1.1.4.5.1 1.1.4.5.2
		Linear interpolation and extrapolation	1.5.4
		Trigonometry	1.5.3.2 1.6.3.1 1.6.3.2 1.6.3.3 1.6.3.4 1.6.3.5 1.6.3.6
		Infinite process	1.8.1.1 1.8.1.2 1.8.1.3 1.8.1.4 1.8.1.5 1.8.1.6 1.8.1.7
		Change	1.8.2.1 1.8.2.2 1.8.2.3 1.8.2.4 1.8.2.5
		Validation	1.9.1.1 1.9.1.2 1.9.1.3 1.9.1.4 1.9.1.5 1.9.1.6 1.9.1.7 1.9.1.8 1.9.1.9
		Structuring	1.9.2.1 1.9.2.2 1.9.2.3 1.9.2.4 1.9.2.5 1.9.2.6 1.9.2.7 1.9.2.8 1.9.2.9
	Other topics	Other topics	1.10.1 1.10.2 1.10.3 1.10.4 1.10.5 1.10.6

NOTE: The header "Original framework codes" refers to the codes assigned to the more than 200 mathematics topics listed in the TIMSS mathematics curriculum framework.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

CHART A3. Ranked aggregation of the Trends in International Mathematics and Science Study (TIMSS) mathematics curriculum framework performance expectations codes for the Mathematics Curriculum Study: 2005

Group rank	Group label	Original framework codes
1 (Lowest)	Definition and computation	2.1.1 2.1.2 2.1.3 2.2.1 2.2.2 2.5.1
2	Estimating, using and representing data	2.2.3 2.5.2 2.5.3
3	Formulating problems and critiquing	2.3.1 2.3.5 2.4.1 2.5.4
4 (Highest)	Problem solving, advanced reasoning, justifying, and proving	2.3.2 2.3.3 2.3.4 2.4.2 2.4.3 2.4.4 2.4.5 2.4.6

NOTE: The header "Original framework codes" refers to the codes assigned to the more than 20 performance expectations listed in the TIMSS mathematics curriculum framework.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

TABLE A1. Course-level factor loadings for algebra I and geometry mathematics topics, by mathematics topic grouping: 2005

Mathematics topic groupings	Algebra I					Geometry					
	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
Arithmetic	-0.015	0.063	0.011	-0.733	0.207	-0.007	0.447	0.673	0.378	-0.209	-0.063
Pre-equation	-0.194	-0.755	-0.017	0.015	0.176	0.096	-0.064	-0.078	0.291	0.842	-0.153
Basic number theory	-0.013	-0.779	-0.023	-0.100	0.143	-0.088	0.754	0.341	0.183	0.014	0.085
Basic equations	-0.303	-0.277	-0.688	-0.157	-0.248	0.138	0.126	0.378	0.840	0.104	-0.037
Advanced equations	-0.247	0.088	0.025	0.044	-0.784	0.340	0.511	-0.106	-0.132	0.217	0.535
Basic functions	-0.184	0.045	0.773	0.169	-0.194	0.668	0.082	0.099	0.259	-0.469	0.092
Advanced functions	-0.110	0.258	0.078	0.146	-0.515	0.723	0.283	-0.104	-0.061	0.334	0.068
Advanced number theory	-0.433	0.050	-0.117	0.390	0.503	0.815	0.098	0.033	0.198	0.027	-0.235
Pre-geometry	0.500	-0.419	-0.108	-0.188	0.213	-0.277	-0.712	0.109	-0.111	0.028	0.276
Two-dimensional geometry	0.681	0.110	0.210	-0.077	0.197	-0.779	-0.021	0.064	-0.190	0.090	-0.486
Three-dimensional geometry	0.786	0.143	-0.030	0.218	0.120	-0.189	-0.108	0.185	-0.672	-0.191	0.315
Coordinate geometry	-0.314	0.417	-0.606	-0.093	0.178	-0.074	0.070	-0.822	0.061	0.033	0.149
Vectors, transformation, congruence, and similarity	0.766	0.125	0.114	0.203	-0.018	-0.211	-0.231	-0.573	-0.363	-0.506	-0.193
Data representation and analysis	0.264	0.583	0.418	-0.172	0.326	0.880	0.077	0.115	0.135	0.064	-0.071
Uncertainty and probability	-0.016	0.022	0.277	0.621	0.449	0.570	0.395	0.049	-0.257	0.462	-0.079
Advanced mathematics[1]	0.294	0.095	0.106	0.671	-0.036	-0.035	-0.155	-0.119	-0.183	-0.168	0.805
Other topics	0.245	0.511	0.594	-0.366	-0.026	0.658	-0.352	0.204	-0.073	0.235	0.226
Percentage of topic variance explained	15.6	13.8	12.6	11.3	10.4	24.0	12.0	11.0	10.9	10.3	9.2

[1] Advanced mathematics is a combination of mathematics topics generally not associated with algebra and geometry. They are generally taught in later mathematics courses, although elements may be taught in algebra and geometry courses. These topics include discrete mathematics, linear interpolation and extrapolation, trigonometry, calculus, and validation and structuring.

NOTE: This table provides course-level factor loadings for algebra I and geometry mathematics topics by mathematics topic groupings. Principal components analyses were used to extract the factor loadings, with a varimax rotation utilizing Kaiser normalization. After 11 rotations, a five factor solution explained 64 percent of the variance for the algebra I topics. After 29 rotations, a six factor solution explained 77 percent of the variance for the geometry topics.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

TABLE A2. Student-level factor loadings for algebra I and geometry mathematics topics, by mathematics topic grouping: 2005

Mathematics topic groupings	Algebra I					Geometry					
	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
Arithmetic	-0.134	0.163	-0.011	-0.792	0.096	0.109	0.867	-0.048	-0.176	-0.167	0.230
Pre-equation	-0.035	-0.217	-0.802	0.147	0.100	-0.051	-0.082	0.119	0.899	-0.139	0.000
Basic number theory	-0.066	-0.116	-0.836	-0.091	0.105	-0.109	0.739	0.376	-0.055	-0.153	-0.019
Basic equations	-0.261	-0.764	-0.187	-0.176	-0.213	0.330	0.738	-0.272	0.446	-0.035	-0.001
Advanced equations	-0.331	0.055	0.188	0.030	-0.763	0.084	0.136	0.787	-0.052	0.335	-0.072
Basic functions	-0.215	0.749	0.055	0.239	-0.292	0.812	0.224	-0.114	-0.295	-0.028	0.066
Advanced functions	-0.130	-0.126	0.384	-0.025	-0.430	0.558	-0.138	0.570	0.293	-0.123	0.052
Advanced number theory	-0.316	-0.161	0.132	0.232	0.723	0.816	0.046	0.237	0.177	-0.256	0.043
Pre-geometry	0.538	-0.213	-0.331	-0.071	0.057	-0.188	-0.340	-0.447	0.093	0.498	0.268
Two-dimensional geometry	0.678	0.372	-0.064	-0.091	0.144	-0.801	-0.043	-0.158	0.000	-0.502	0.112
Three-dimensional geometry	0.823	0.017	0.112	0.210	0.021	-0.310	-0.276	0.101	-0.486	0.308	0.329
Coordinate geometry	-0.129	-0.484	0.498	-0.135	0.294	-0.069	-0.302	0.039	0.073	0.063	-0.834
Vectors, transformation, congruence, and similarity	0.849	0.168	0.179	0.167	-0.059	-0.114	-0.598	-0.286	-0.508	-0.128	-0.358
Data representation and analysis	0.344	0.480	0.449	-0.156	0.263	0.887	0.096	0.257	0.110	-0.074	0.085
Uncertainty and probability	-0.072	0.253	-0.014	0.687	0.439	0.258	0.015	0.795	0.157	-0.174	0.117
Advanced mathematics[1]	0.117	0.199	-0.084	0.675	0.061	-0.024	-0.143	0.039	-0.193	0.844	-0.030
Other topics	0.257	0.653	0.345	-0.438	-0.086	0.511	-0.207	0.175	0.310	0.251	0.482
Percentage of topic variance explained	16.1	14.6	13.6	12.0	10.8	21.7	15.6	13.5	11.3	9.9	7.9

[1] Advanced mathematics is a combination of mathematics topics generally not associated with algebra and geometry. They are generally taught in later mathematics courses, although elements may be taught in algebra and geometry courses. These topics include discrete mathematics, linear interpolation and extrapolation, trigonometry, calculus, and validation and structuring.

NOTE: This table provides student-level factor loadings for algebra I and geometry mathematics topics by mathematics topic groupings. Principal components analyses were used to extract the factor loadings, with a varimax rotation utilizing Kaiser normalization. After 40 rotations, a five factor solution explained 67 percent of the variance for the algebra I topics. After 13 rotations, a six factor solution explained 80 percent of the variance for the geometry topics.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

TABLE A3. Percentage of subject matter content in algebra I and geometry courses, by course category and mathematics topic grouping: 2005

Mathematics topic groupings	Algebra I course category				Geometry course category			
	Low	Medium	High	Integrated	Low	Medium	High	Integrated
Arithmetic	10.57	9.07	7.45	11.99	4.02	3.29	1.69	4.29
Pre-equation	10.12	5.82	3.97	2.36	1.65	1.07	2.11	2.00
Basic number theory	7.57	3.14	2.22	1.53	0.30	0.43	0.07	0.66
Basic equations	28.67	30.67	21.40	13.02	8.29	7.16	6.63	15.78
Advanced equations	10.70	14.63	17.55	4.18	0.68	0.76	0.27	7.41
Basic functions	2.58	2.66	6.80	3.09	0.16	0.33	0.00	3.15
Advanced functions	0.73	2.36	2.07	1.52	0.12	0.08	0.02	2.22
Advanced number theory	6.16	7.03	9.47	3.61	1.16	0.95	0.65	8.94
Pre-geometry	6.81	3.90	1.99	5.72	10.99	9.59	10.06	5.18
Two-dimensional geometry	3.78	2.25	3.31	12.69	43.61	42.02	43.52	11.74
Three-dimensional geometry	0.23	0.10	0.23	2.47	6.54	5.63	5.03	2.82
Coordinate geometry	5.54	7.61	5.89	5.05	2.74	4.03	6.32	4.17
Vectors, transformation, congruence, and similarity	0.47	0.63	0.79	3.91	10.24	13.99	15.29	4.18
Data representation and analysis	3.31	4.34	6.18	15.59	0.32	0.36	0.05	12.81
Uncertainty and probability	1.55	2.24	5.30	3.00	0.53	0.59	0.89	4.06
Advanced mathematics[1]	1.09	2.96	4.33	4.54	8.08	9.00	6.93	7.77
Other	0.12	0.60	1.04	5.72	0.57	0.73	0.46	2.83
Number of courses	173	527	197	93	138	581	213	63

[1] Advanced mathematics is a combination of mathematics topics generally not associated with algebra and geometry. They are generally taught in later mathematics courses, although elements may be taught in algebra and geometry courses. These topics include discrete mathematics, linear interpolation and extrapolation, trigonometry, calculus, and validation and structuring.

NOTE: This table provides the percentage of subject matter content, as defined by the mathematics topic groupings, for algebra I and geometry courses by course category. The percentages were generated by calculating the mean for each of the 17 mathematics topic groupings by the course categories generated from the discriminant analysis. School weights were used to adjust for sampling differences. Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

TABLE A4. Percentage of subject matter content in students' algebra I and geometry courses, by course level and mathematics topic grouping: 2005

Mathematics topic groupings	Algebra I course level				Geometry course level			
	Beginner	Intermediate	Advanced	Integrated	Beginner	Intermediate	Advanced	Integrated
Arithmetic	10.16	9.40	8.18	9.89	3.16	3.33	1.66	4.49
Pre-equation	6.66	3.87	2.19	5.58	10.99	9.73	9.72	5.25
Basic number theory	10.64	5.71	4.19	2.89	1.88	1.08	1.98	1.39
Basic equations	7.53	2.98	2.41	0.76	0.27	0.44	0.07	0.80
Advanced equations	28.09	31.35	20.62	11.02	7.56	7.50	6.34	16.59
Basic functions	11.70	15.17	16.23	4.47	1.05	0.80	0.20	7.05
Advanced functions	2.31	2.45	6.46	4.22	0.13	0.35	0.00	3.17
Advanced number theory	0.43	2.51	1.74	1.62	0.25	0.09	0.02	1.50
Pre-geometry	6.10	6.02	10.73	2.42	1.11	0.95	0.65	9.41
Two-dimensional geometry	4.49	2.16	3.59	13.81	41.56	41.19	43.91	13.54
Three-dimensional geometry	0.25	0.14	0.12	2.12	7.28	5.78	5.21	2.57
Coordinate geometry	4.96	7.54	6.81	5.03	3.31	3.94	6.76	3.61
Vectors, transformation, congruence, and similarity	0.34	0.53	0.54	5.02	10.30	13.90	15.95	3.62
Data representation and analysis	3.08	4.55	6.12	18.08	0.47	0.34	0.06	12.50
Uncertainty and probability	1.48	2.04	5.33	2.12	0.70	0.61	0.80	4.05
Advanced mathematics[1]	1.67	2.86	3.87	3.41	9.21	9.21	6.33	7.54
Other	0.11	0.73	0.88	7.56	0.79	0.77	0.37	2.93

[1] Advanced mathematics is a combination of mathematics topics generally not associated with algebra and geometry. They are generally taught in later mathematics courses, although elements may be taught in algebra and geometry courses. These topics include discrete mathematics, linear interpolation and extrapolation, trigonometry, calculus, and validation and structuring.

NOTE: This table provides the percentage of subject matter content, as defined by the mathematics topic groupings, for algebra I and geometry courses by course level. The percentages were generated by calculating the mean for each of the 17 mathematics topic groupings by the course levels generated from the discriminant analysis. School weights were used to adjust for sampling differences. Details may not sum to total because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, High School Transcript Study (HSTS), Mathematics Curriculum Study, 2005.

Glossary

Advanced algebra

A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study to describe mathematics topics that cover the more complex elements of algebra. These topics include advanced equations, basic function, advanced functions, and advanced number theory.

Advanced geometry

A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study and includes mathematics topics that cover advanced geometric concepts. These topics include three-dimensional geometry, coordinate geometry, and vector geometry.

Beginner course level

The course level that indicates a student's coursework covers more introductory material and less advanced material than intermediate level courses.

Chapter summary measures

Statistical measures used to summarize the subject matter content and cognitive challenge of a chapter in a textbook. The measures include the total content page count and a set of weighted topic content page counts.

Cognitive challenge

The complexity of student tasks that are required to answer the chapter review questions in a textbook. It is measured by the performance expectations.

Course categories

Empirically derived categories of algebra I or geometry courses based on the combination of content and challenge of the course, as determined by the textbooks used. There are four course categories: low, medium, high, and integrated.

Course level

A ranking of a student's algebra I or geometry courses based on the combination of curriculum topics covered and the level of challenge of the courses, as determined by the content of their textbooks. There are three course levels: beginner, intermediate, and rigorous.

Course summary measures

Statistical measures used to summarize the subject matter content and cognitive challenge of an algebra I or geometry course. The measures include the total content page count, the overall cognitive challenge rating, and a set of weighted topic content page counts.

Curriculum topics

Six broad categories of mathematics topics used to present results of the 2005 Mathematics Curriculum Study. The six curriculum topic categories that are covered in algebra I and geometry courses are elementary and middle school mathematics, introductory algebra, advanced algebra, two-dimensional geometry, advanced geometry, and other high school mathematics.

Elementary and middle school mathematics

A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study to describe mathematics topics that are traditionally taught before a student takes an algebra I course. These topics include elements of basic arithmetic and pre-geometry.

High course category

A course category that indicates that the content and challenge of an algebra I or geometry course most closely resembled year two of a two-year algebra I course or an honors geometry course, respectively.

"Honors" algebra I course

An algebra I course described by the school as covering more advanced algebra topics and/or more in-depth analysis of algebra topics than a "regular" algebra I course, including courses labeled honors, gifted and talented, and college preparatory.

"Honors" geometry course

A geometry course described by the school as covering more advanced geometry topics and/or more in-depth analysis of geometry topics than a "regular" geometry course, including courses labeled honors, gifted and talented, and college preparatory.

"Informal" geometry course

A geometry course described by the school as de-emphasizing the need for proofs.

Integrated mathematics course

A mathematics course that covers several mathematics topics or strands, such as algebra, geometry, trigonometry, statistics, and analysis, in one course.

Intermediate course level

The course level that indicates a student's coursework contains a balance of both introductory and advanced material.

Introductory algebra

A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study and includes mathematics topics needed to understand the basics of algebra and provide the foundation for learning advanced algebra. These topics include pre-algebra, basic algebraic equations, and basic number theory.

Low course category

A course category that indicates that the content and challenge of an algebra I or geometry course most closely resembled year one of a two-year algebra I course or an informal geometry course, respectively.

Medium content category

A course category that indicates that the content and challenge of an algebra I or geometry course most closely resembled a regular algebra I or geometry course, respectively.

Other high school mathematics

A broad curriculum topic category that is used in the 2005 Mathematics Curriculum Study and includes mathematics topics that are traditionally taught in courses taken after geometry and algebra II. These topics include trigonometry, pre-calculus, statistics, validation and structuring, discrete mathematics, finite mathematics, and calculus.

Overall cognitive challenge rating

A summary measure that indicates the overall complexity of the student tasks that are required to answer the chapter review questions in a textbook. It is calculated by summing the weighted topic content page counts across all topic groupings and dividing the sum by the total content page count.

Performance expectations

The activities or skills a student is expected to use to correctly answer a chapter review question.

"Regular" algebra I course

An algebra I course described by the school as the course students take when progressing through the school's standard mathematics sequence.

"Regular" geometry course

A geometry course described by the school as the course students takes when progressing through the school's standard mathematics sequence.

Rigorous course level

The course level that indicates a student's coursework covers more advanced material and less introductory material than intermediate level courses.

Student summary measures

Statistical measures used to summarize the subject matter content and cognitive challenge of a student's coursework in algebra I or geometry. The measures include the total content page count, the overall cognitive challenge rating, and a set of weighted topic content page counts.

Total content page count

A summary measure that represents the total amount of instructional material in a textbook chapter, algebra I or geometry course, or student coursework.

Two-dimensional geometry

A broad curriculum topic that is used in the 2005 Mathematics Curriculum Study and includes mathematics topics that focus on basic linear and planar geometric concepts. These topics include basic geometric concepts (e.g., points, angles, parallelism, and perpendicularity) and the properties of shapes.

"Two-year" algebra I course

An algebra I course described by the school as taking two school years to complete.

Weighted topic content page count

A summary measure that represents the amount of instructional material within a textbook chapter, algebra I or geometry course, or student coursework devoted to a mathematics topic, as weighted by ratings of the performance expectation codes assigned to the chapter review questions for that mathematics topic.

References

Chaney, B., Burgdorf, K., and Atash, N. (1997). Influencing Achievement Through High School Graduation Requirements. *Educational Evaluation and Policy Analysis, 19*(3): 229–244.

Cogan, L.S., Schmidt, W.H., and Wiley, D.E. (2001). Who Takes What Math and in Which Track? Using TIMSS to Characterize U.S. Students' Eighth-Grade Mathematics Learning Opportunities. *Educational Evaluation and Policy Analysis, 23*(4): 323–341.

Gamoran, A., Porter, A.C., Smithson, J., and White, P. A. (1997). Upgrading High School Mathematics Instruction: Improving Learning Opportunities for Low-Achieving, Low-Income Youth. *Educational Evaluation and Policy Analysis, 19*(4): 325–338.

Grigg, W., Donahue, P., and Dion, G. (2007). *The Nation's Report Card: 12th-Grade Reading and Mathematics 2005* (NCES 2007-468). U.S. Department of Education, National Center for Education Statistics. Washington, D.C.: U.S. Government Printing Office.

Krewski, D., and Rao, J. (1981). Inference From Stratified Samples: Properties of Linearization, Jackknife, and Balanced Replication Methods. *The Annals of Statistics 9*: 1010–1019.

Medrich, E., Brown, C., Henke, R., and Ross, L. (1992). *Overview and Inventory of State Requirements for School Coursework and Attendance* (NCES 92-663). U.S. Department of Education, National Center for Education Statistics. Washington, DC: U.S. Government Printing Office.

Muller, C., Pearson, J., Riegle-Crumb, C., Requejo, J.H., Frank, K.A., Schiller, K., Raley, R.K., Langenkamp, A.G., Crissey, S., Mueller, A.S., Callahan, R., Wilkinson, L., and Field, S. (2007). *National Longitudinal Study of Adolescent Health: Wave III Education Data.* Chapel Hill: Carolina Population Center, University of North Carolina at Chapel Hill.

Peak, L. (1996). *Pursuing Excellence: A Study of U.S. Eighth-Grade Mathematics and Science Teaching, Learning, Curriculum, and Achievement in International Context. Initial Findings From the Third International Mathematics and Science Study* (NCES 97-198). Washington D.C.: National Center for Education Statistics, Office of Educational Research and Improvement.

Robitaille, D.F., McKnight, C., Schmidt, W.H., Britton, E., Raizen, S., and Nicol, C. (1993). Curriculum Frameworks for Mathematics and Science: TIMSS Monograph No.1. Vancouver, B.C.: Pacific Educational Press.

Schiller, K.S., Muller, C., Schmidt, W.H., Houang, R., Requejo, J.H., Riegle-Crumb, C., Pearson, J., and Frank, K. (2008). *National Longitudinal Study of Adolescent Health (Add Health), Wave III Education Data, Student-Level Curriculum Component: Math & Science Textbook Study.* Chapel Hill: North Carolina Population Center, University of North Carolina at Chapel Hill.

Schiller, K., Schmidt, W., Muller, C., and Houang, R. (2010). Hidden Disparities: How Courses and Curricula Shape Opportunities in Mathematics During High School. *Equity and Excellence in Education, 43*(4).

Schmidt, W.H., McKnight, C.C., and Raizen, S.A. (1997). *A Splintered Vision: An Investigation of U.S. Science and Mathematics Education.* Boston: Kluwer Academic Publishers.

Schmidt, W.H., McKnight, C.C., Valverde, G.A., Houang, R.T., and Wiley, D.E. (1997). *Many Visions, Many Aims: A Cross-National Investigation of Curricular Intentions in School Mathematics.* Dordrecht, The Netherlands: Kluwer.

Schmidt, W.H., McKnight, C.C., Houang, R.T., Wang, H., Wiley, D.E., Cogan, L.S., and Wolfe, R.G. (2001). *Why Schools Matter: Examining Cross-National Comparisons of Curriculum and Learning.* San Francisco: Jossey-Bass.

Schmidt, W.H. (2012). Measuring Content Through Textbooks: The Cumulative Effect of Middle-School Tracking. In G. Gueudet, B. Pepin, and L. Trouche (Eds.), *From Text to "Lived" Resources: Mathematics Curriculum Materials and Teacher Development* (Mathematics Teacher Education, Vol. 7) (pp. 143-160). Springer: Dordrecht

Shettle, C., Roey, S., Mordica, J., Perkins, R., Nord, C., Teodorovic, J., Brown, J., Lyons, M., Averett, C., and Kastberg, D. (2007). *The Nation's Report Card: America's High School Graduates* (NCES 2007-467). U.S. Department of Education, National Center for Education Statistics. Washington, DC: U.S. Government Printing Office.

Shettle, C., Cubell, M., Hoover, K., Kastberg, D., Legum, S., Lyons, M., Perkins, R., Rizzo, L., Roey, S., and Sickles, D. (2008). *The 2005 High School Transcript Study: The 2005 High School Transcript Study User's Guide and Technical Report* (NCES 2009-480rev). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Washington, DC.

Stevenson, D.L. and Schiller, K.S. (1999). State Education Policies and Changing School Practices: Evidence From the National Longitudinal Study of Schools, 1980–1993. *American Journal of Education, 107*: 261–288.

Tornroos, J. (2005). Mathematics Textbooks, Opportunity to Learn, and Student Achievement. *Studies in Educational Evaluation, 31*(4): 315–327.

U.S. DEPARTMENT OF EDUCATION

The National Assessment of Educational Progress (NAEP) is a congressionally authorized project sponsored by the U.S. Department of Education. The National Center for Education Statistics, within the Institute of Education Sciences, administers NAEP. The Commissioner of Education Statistics is responsible by law for carrying out the NAEP project.

"The Department of Education's mission is to promote student achievement and preparation for global competitiveness by fostering educational excellence and ensuring equal access."

THE NATIONAL ASSESSMENT GOVERNING BOARD

In 1988, Congress created the National Assessment Governing Board to set policy for the National Assessment of Educational Progress, commonly known as The Nation's Report Card™. The Governing Board is an independent, bipartisan group whose members include governors, state legislators, local and state school officials, educators, business representatives, and members of the general public.

CONTENT CONTACT

Janis Brown
202-502-7482
Janis.Brown@ed.gov

MORE INFORMATION

The report release site is http://nationsreportcard.gov. The NCES web electronic catalog is http://nces.ed.gov/pubsearch.

For ordering information, write to
ED Pubs
U.S. Department of Education
P.O. Box 22207
Alexandria, VA 22304

or call
toll free 1-877-4ED-Pubs

or order online at
http://www.EDPubs.gov.

This report was prepared in part under Contract No. ED-07-CO-0079 with Westat. Mention of trade names, commercial products, or organizations does not imply endorsement by the U.S. Government.

March 2013

SUGGESTED CITATION

Brown, J., Schiller, K., Roey, S., Perkins, R., Schmidt, W., and Houang, R. (2013). *Algebra I and Geometry Curricula* (NCES 2013-451). U.S. Department of Education, National Center for Education Statistics. Washington, DC: U.S. Government Printing Office.