NAEP
NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

ies NATIONAL CENTER FOR EDUCATION STATISTICS
Institute of Education Sciences

School Composition and the Black–White Achievement Gap: Methodology Companion

NCES 2015-032

U.S. DEPARTMENT OF EDUCATION

U.S. Department of Education
Arne Duncan
Secretary

Institute of Education Sciences
Sue Betka
Acting Director

National Center for Education Statistics
Peggy G. Carr
Acting Commissioner

Assessment Division
Peggy G. Carr
Associate Commissioner

The National Center for Education Statistics (NCES) is the primary federal entity for collecting, analyzing, and reporting data related to education in the United States and other nations. It fulfills a congressional mandate to collect, collate, analyze, and report full and complete statistics on the condition of education in the United States; conduct and publish reports and specialized analyses of the meaning and significance of such statistics; assist state and local education agencies in improving their statistical systems; and review and report on education activities in foreign countries.

NCES activities are designed to address high-priority education data needs; provide consistent, reliable, complete, and accurate indicators of education status and trends; and report timely, useful, and high-quality data to the U.S. Department of Education, the Congress, the states, other education policymakers, practitioners, data users, and the general public. Unless specifically noted, all information contained herein is in the public domain.

We strive to make our products available in a variety of formats and in language that is appropriate to a variety of audiences. You, as our customer, are the best judge of our success in communicating information effectively. If you have any comments or suggestions about this or any other NCES product or report, we would like to hear from you. Please direct your comments to

National Center for Education Statistics
Institute of Education Sciences
U.S. Department of Education
1990 K Street, NW
Washington, DC 20006-5651

June 2015

The NCES Home Page address is http://nces.ed.gov.
The NCES Publications and Products address is http://nces.ed.gov/pubsearch.

This publication is only available online. To download, view, and print the report as a PDF file, go to the NCES Publications and Products address shown above.

This report was prepared to support the NCES report *School Composition and the Black–White Achievement Gap* for the National Center for Education Statistics under Contract No. ED-04-CO-00025/0019 with the American Institutes for Research™. Mention of trade names, commercial products, or organizations does not imply endorsement by the U.S. Government.

Suggested Citation
Bohrnstedt, G., Kitmitto, S., Ogut, B., Sherman, D., and Chan, D. (2015). *School Composition and the Black–White Achievement Gap: Methodology Companion* (NCES 2015-032). U.S. Department of Education, Washington, DC: National Center for Education Statistics. Retrieved [date] from http://nces.ed.gov/pubsearch.

Full results can be found at http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2015032.

Content Contact
Taslima Rahman
(202) 502-7316
taslima.rahman@ed.gov

Contents

This page intentionally left blank.

Introduction

The School Composition and the Black–White Achievement Gap study was undertaken by the National Center for Education Statistics to present both descriptive and associative information on the relationships among the percentage of students in a school who were Black[1] (referred to as "Black student density" or "density"), the Black–White student academic performance gap, and student achievement. The study used data from the 2011 Grade 8 mathematics administration of the National Assessment of Educational Progress (NAEP), along with data from the Common Core of Data (CCD).[2] This methodology companion provides details and technical documentation for the two chapters of the report that use statistical analysis, namely those on (a) the relationship between Black student density and achievement and (b) the decomposition of the overall gap into between-school and within-school components.

Methodology for "The Relationship Between Black Student Density and Achievement" Chapter

As part of the methodology, control variables derived from NAEP student and school survey questionnaire items were used in a statistical model to isolate the relationship between density and achievement. Direct estimation procedures[3] employed to estimate relationships allowed models to incorporate variables in the analysis that were not included as part of the statistical model used to compute the plausible values associated with NAEP. Because NAEP analyses estimate population values for a group of students using their responses to cognitive items without computing scores for each student, plausible values that are random draws from these population values are created for secondary data users. These plausible values allow relatively straightforward computations of statistics, such as regression coefficients, that account for some of the uncertainty in individual test scores.[4] In particular, the key interaction in the model used for this report—the interaction between density (percentage of students in the school who were Black) and student race/ethnicity (Black or White)—was not included in the population-structure model. The direct estimation method used in these analyses employs a Taylor series expansion that computes robust standard errors for estimates for complex survey data where observations are clustered with an unequal probability of selection.[5]

Data

This chapter uses data from the 2011 NAEP Grade 8 mathematics assessment. Because the report focused on the gap between Black and White students, the data were limited to those students, which left approximately 117,100 public school students out of about 175,200 in the total NAEP sample. The sample was further reduced to approximately 96,910 students by listwise deletion of the data for which one or more of the model covariates were missing.[6,7] The report focused mainly on *Black student density*, defined as "the percentage of students in a student's school who are Black." The following density categories were used to

[1] The category Black includes students who identified as "Black or African American."
[2] https://nces.ed.gov/ccd/.
[3] For direct estimation, the AM software was used. AM is a statistical software package for analyzing data from complex samples, especially large-scale assessments like NAEP. Available at http://am.air.org/.
[4] As stated on the NAEP technical documentation on the Web: ". . . when a group-defining variable is not included in the population-structure model, mean scores for the groups defined by that variable, based on plausible values, may or may not be good estimates of the group means." From "Plausible Values Versus Individual Scores," *NAEP Technical Documentation on the Web*. Retrieved 1/20/2014 from http://nces.ed.gov/nationsreportcard/tdw/analysis/est_pv_individual.asp.
[5] http://am.air.org/help/NAEPTextbook/htm/otaylorseriesexpansion.htm.
[6] The listwise deletion method for handling missing data was chosen for two reasons: first, there was little support for imputing values because we discovered that when one piece of information was missing other pieces of information needed for the imputation also were often missing (because of non-response), especially for the teachers' questionnaire; and second, the direct estimation methods used here are computationally complex making methods for handling missing data, such as multiple imputation, difficult to implement.
[7] The comparisons between the listwise deleted sample and the full sample were conducted for student achievement and other variables, including proportion of Black students. The results showed that the differences between the two samples were .04 or smaller in effect size for the core variables (see table 2) and between .00 and .10 for the teacher qualifications, teacher strategies, and school resources (see tables 3, 4, and 5). These results suggest no substantial differences between the listwise deleted sample and the full sample.

indicate the percentage of Black students in a school: 0–20%, 20–40%, 40–60%, and 60–100%.[8] There were about 4,150 schools in 0–20% category in contrast to 520 schools that had 60–100% Black students. Table 1 shows the sample distribution by race/ethnicity and density category.

Table 1. Sample size, by race/ethnicity and density category

Group	Density category			
	0–20%	20–40%	40–60%	60–100%
Students				
White	65,830	6,630	2,280	740
Black	4,030	4,270	3,460	9,690
Schools	4,150	580	280	520

NOTE: Sample sizes are rounded to the nearest 10.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2011 Mathematics Grade 8 Assessment.

Model Development

In specifying the model, a large number of NAEP background items as control variables initially were included in the models, but direct estimation computations regularly failed to converge, primarily because of high collinearity among some of these variables. Hence, rather than attempting to include all potentially relevant covariates in the models, a set of core variables that were necessary for the intended analysis (density and the Black student indicator) or were identified by previous literature[9] as strong correlates of achievement—for example, student socioeconomic status (SES) variables and school SES variables—were selected. The next step was a model-building process, in which the list of other covariates selected for the final model was narrowed down. The model-building process included the following three stages, which are explained in detail in the following sections of this document but are summarized here:[10,11]

- **Stage 1—Initial variable selection, using plausible values procedure.** NAEP plausible values were used as the dependent variables to identify covariates for potential inclusion in the model. Criteria included having a statistically significant association with NAEP Grade 8 mathematics achievement, and an association for White students different from that of Black students.
- **Stage 2—Secondary variable selection using direct estimation.** Direct estimation was used to examine whether statistically significant covariates identified in Stage 1 continued to be statistically significant when this more rigorous form of estimation is used.
- **Stage 3—Final estimation using direct estimation.** Direct estimation was used to estimate the final model, which included core variables, plus other covariates that retained statistical significance in Stage 2.

The variables necessary for estimating the relationship between achievement, the achievement gap, and density included the following (see also table 2):

- An indicator for whether the student was Black

[8] In these analyses, the categories of percentage Black and other variables that were categorized are constructed such that there is no overlap between the levels. For example, the first density category is 0% up to and equal to 20%, the second is greater than 20% up to and equal to 40%, and so forth.
[9] See for example Cheema and Galuzzo (2013), Harwell and LeBeau (2010), and Sirin (2005).
[10] For model building, interest was in each variable individually; hence aggregates of multiple potential control variables were not created.
[11] Because the data analyzed in this report were limited to what are collected by NAEP, there may be factors relevant to the Black-White achievement gap that were not measured by, or not adequately measured by, variables used in the model.

- Black student density, which was measured categorically, using indicators for schools' density ranges:
  - 0–20% Black
  - 20–40% Black
  - 40–60% Black
  - 60–100% Black

Originally, density was categorized into five evenly sized categories, each covering 20 percentage points. However, there were relatively few observations in the 60–80% and 80–100% categories, so they were combined.

The following core variables were chosen to be included in the model because of their strong support in the literature[12] as correlates of achievement and achievement gaps (see table 2).

- Individual Student characteristics:
  - A student-level indicator for having a disability
  - Indicator for being male
- SES measures at the student level:
  - Indicator for National School Lunch Program (NSLP) eligibility
  - Indicators for mother's and father's highest level of education
  - An indicator for having more than 26 books in the home[13]
  - An indicator for the absence of an encyclopedia in the home
- SES measures at the school level:
  - Percentage of students in the school who are NSLP eligible
  - Percentage of NAEP Grade 8 sample with parent's combined highest level of education[14] unknown
  - Percentage of NAEP Grade 8 sample with parent's combined highest level of education being high school or some education after high school
  - Percentage of NAEP Grade 8 sample with parent's combined highest level of education being college graduate or higher
  - Percentage of NAEP Grade 8 sample with more than 26 books in the home
  - Percentage of NAEP Grade 8 sample without an encyclopedia in the home

Each stage of the model-building process for choosing other variables to include is described in further detail below.

[12] See for example Cheema and Galuzzo (2013), Harwell and LeBeau (2010), and Sirin (2005).
[13] Going back as early as the work of Chapin (Chapin 1933) and Sewell (Sewell 1940), sociologists have used household possessions as proxies for SES and these measures are currently still in use (e.g., Konstantopoulos and Hedges 2008). However, because the validity of a given household item as a measure of SES can change over time NCES has recently removed and added new items to its list. NCES is also studying other ways to improve the measurement of SES in NAEP (U.S. Department of Education 2012).
[14] The combined highest level of education was determined by taking the maximum of the two separate parent education levels.

Table 2. Core variables, their source, and whether their interaction with Black student indicator was included in the analysis

Variable description	NAEP questionnaire	Interaction included
Black	Student	No
Male	Student	No
Student with mother's highest education unknown	Student	No
Student with mother's highest education ≥ HS	Student	No
Student with mother's highest education ≥ BA	Student	No
Student with father's highest education unknown	Student	No
Student with father's highest education ≥ HS	Student	No
Student with father's highest education ≥ BA	Student	No
NSLP eligible	Student	No
Individualized Education Program (IEP) student	Student	No
Student's home has ≥ 26 books	Student	No
Student's home has an encyclopedia = No	Student	No
Student's home has an encyclopedia = Missing value	Student	No
Grade 8: Weighted proportion male	Derived from student	No
Grade 8: Weighted proportion of students with parents' highest education unknown	Derived from student	No
Grade 8: Weighted proportion of students with parents' highest education ≥ HS	Derived from student	No
Grade 8: Weighted proportion of students with parents' highest education ≥ BA	Derived from student	No
Grade 8: Weighted proportion of students who are NSLP eligible	Derived from student	No
Grade 8: Weighted proportion of students who are in an IEP	Derived from student	No
School-level weighted proportion of students with ≥ 26 books at home	Derived from student	No
School-level weighted proportion of students with encyclopedia = No	Derived from student	No
School-level weighted proportion of students with encyclopedia = Missing value	Derived from student	No
Density: 20–40%	Derived from student	Yes
Density: 40–60%	Derived from student	Yes
Density: 60–100%	Derived from student	Yes

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2011 Mathematics Grade 8 Assessment.

Stage 1—Initial variable selection, using plausible values
A wide range of covariates that theoretically, on the basis of previous literature, could affect achievement and/or the achievement gap were considered. The variables were classified into three categories, from which we intended to select at least one measure:

- Teacher qualifications (from the NAEP teacher questionnaire)
- Teacher instructional strategies (also from the NAEP teacher questionnaire)
- School resources and climate (from the NAEP school questionnaire)

To explore which variables to keep in the model, linear regression models were estimated, using student achievement as the dependent variable, the core independent variables listed above (student characteristics, student and school SES measures, and the Black student density measure), and variables from each of the above three categories where each category was considered individually (teacher qualifications, or teacher instructional strategies, or school resources and climate). Stage 1 exploratory models were estimated using NAEP plausible values.[15] The model also included interaction terms between each of the independent variables in the category being explored and the Black student indicator to determine whether each of these variables had differential associations with student achievement for Black students and White students (i.e., contributed to or diminished the achievement gap).[16] From these results, variables that were estimated to have a statistically significant ($p < .05$) or near significant ($p < .10$) differential association for Black versus White students were selected for further exploration in the next stage. Because the goal was to control for factors that might explain the achievement gap, variables were deemed important to be included in the model if they might have a differential impact on Black students than on White students.

Stage 2—Secondary variable selection using direct estimation
Stage 2 models were the same as Stage 1 models, with two exceptions. First, the estimated regression models included only the variables from each of the three categories that were statistically significant ($p < .05$) or near significant statistically ($p < .10$) when assessing differential effects for Black versus White students. Second, the estimation was conducted using direct estimation rather than standard regression analysis using plausible values. Within each set, we retained for inclusion in the final, Stage 3, model variables and their interactions that were statistically significant using direct estimation.[17]

Stage 3—Final estimation using direct estimation
The final model included our core student characteristics, our core student and school SES measures, and the following control variables derived from our Stage 1 and Stage 2 analyses:

- From the teacher qualifications category: indicators for whether the teacher had a math major (undergraduate or graduate degree), a math minor (undergraduate or graduate degree), or no math higher education degree
- From the instructional strategies category: indicators for the extent to which the teacher used different methods of instruction for different students—none to small, moderate, or a large number of different methods—and whether the teacher assigned more than 1 hour of homework a night
- From the school factors category: indicators for the extent of student absences (representing a school climate indicator) and the student/teacher ratio (representing a school resources indicator).

[15] Early attempts to explore all potential covariates jointly in the same model failed to converge using direct estimation methods, likely because of the high degree of collinearity between some of the measures. Hence, we explored variables in the categories described above using plausible values rather than direct estimation to ensure that we were able to obtain results.
[16] Our set of "core" independent variables was not interacted with the Black student indicator.
[17] The only exception to this rule was the removal of the variable "Kind of calculator used during math lessons—Graphics," because AM models did not converge when this variable was included among the independent variables.

Tables 3 to 5 display all the control variables that were considered, their sources, whether their main effects and interactions with the Black variable were significant, and whether they were included in the final estimation model.

Table 3. Teacher qualifications variables considered for use in the estimation models, and their source, significance, and inclusion in the final model

Variable	NAEP questionnaire	Interaction significance in Stage 1	Interaction significance in Stage 2	Included in final model
Years of teaching (5–9)	Teacher	No	No	No
Years of teaching (10–19)	Teacher	No	No	No
Years of teaching (20 or more)	Teacher	No	No	No
Certification type—Other	Teacher	No	No	No
Degree (BA or higher)	Teacher	No	No	No
Degree (undergrad and grad combined math major)	Derived from teacher	No	+	Yes
Degree (undergrad and grad combined math minor)	Derived from teacher	+	No	Yes
Leader for mathematics education	Teacher	No	No	No
Teacher race—Black	Teacher	No	No	No
Teacher race—Other	Teacher	No	No	No
Alternative teaching certificate	Teacher	No	No	No
Not designated as high quality	Teacher	No	No	No
Certified by the National Board for Professional Teaching Standards (NBPTS) in at least one content area	Teacher	No	No	No
Participates in adequate yearly progress improvement & state accountability—No	Teacher	No	No	No

+ Statistically significant ($p < .10$).
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2011 Mathematics Grade 8 Assessment.

Table 4. Teacher strategies variables considered for use in the estimation models, and their source, significance, and inclusion in the final model

Variable	NAEP questionnaire	Interaction significance in Stage 1	Interaction significance in Stage 2	Included in final model
Instructional hours (5–7)	Teacher	No	No	No
Instructional hours (7 or more)	Teacher	No	No	No
Extent uses additional materials—Large	Teacher	No	No	No
Extent uses additional materials—Moderate	Teacher	No	No	No
Adjusts teaching strategy—Daily	Teacher	No	No	No
Adjusts teaching strategy—Monthly	Teacher	No	No	No
Adjusts teaching strategy—Weekly	Teacher	No	No	No
Heavy emphasis on algebra	Teacher	No	No	No
Heavy emphasis on analysis	Teacher	+	No	No

See notes at end of table.

Table 4. Teacher strategies variables considered for use in the estimation models, and their source, significance, and inclusion in the final model—Continued

Variable	NAEP questionnaire	Interaction significance in Stage 1	Interaction significance in Stage 2	Included in final model
Kind of calculator used during math lessons—Graphics	Teacher	*	NC	No
Kind of calculator used during math lessons—Scientific	Teacher	No	No	No
Uses calculator for math tests—Always	Teacher	No	No	No
Use calculator for math tests—Some	Teacher	No	No	No
Engages some students in different activities—Moderate and large	Teacher	No	No	No
Extent uses different methods for different students—Large	Teacher	No	No	Yes
Extent uses different methods for different students—Moderate	Teacher	+	*	Yes
Extent changes pace for some students—Large	Teacher	No	No	No
Extent changes pace for some students—Moderate	Teacher	No	No	No
Extent uses different standards for different students—Large	Teacher	No	No	No
Extent uses different standards for different students—Moderate	Teacher	No	No	No
Extent uses different standards for different students—Small	Teacher	No	No	No
Emphasis on geometry—Heavy	Teacher	No	No	No
Daily homework given >1 hour	Teacher	*	+	Yes
Assesses math with multiple-choice tests—Monthly	Teacher	No	No	No
Assesses math with multiple-choice tests—Weekly	Teacher	No	No	No
Emphasis on measurement—Heavy	Teacher	No	No	No
Emphasis on measurement—Moderate	Teacher	No	No	No
Emphasis on numbers and operations—Heavy	Teacher	No	No	No
Assesses math with problem sets—Weekly	Teacher	No	No	No
Extent uses project work in instruction—Weekly or monthly	Teacher	No	No	No
Sets goals for specific programs—Monthly	Teacher	No	No	No
Sets goals for specific programs—Daily and weekly	Teacher	No	No	No
Discusses progress toward goal—Monthly	Teacher	No	No	No
Discusses progress toward goal—Daily and weekly	Teacher	No	No	No
Assesses math with short/long written responses—Monthly	Teacher	No	No	No
Assesses math with short/long written responses—Weekly	Teacher	No	No	No

* Statistically significant ($p < .05$).
+ Statistically significant ($p < .10$).
NOTE: NC indicates model with this variable did not converge in AM.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2011 Mathematics Grade 8 Assessment.

Table 5. School resources and climate variables considered for use in the estimation models, and their source, significance, and inclusion in the final model—Continued

Variable	NAEP questionnaire	Interaction significance in Stage 1	Interaction significance in Stage 2	Included in final model
Log of school-level enrollment	School	No	No	No
Receives Title I funding	School	No	No	No
Number of regularly scheduled volunteers—1–3	School	No	No	No
Number of regularly scheduled volunteers—4 or more	School	No	No	No
Percentage of students absent per day—3–5	School	No	No	Yes
Percentage of students absent per day—6 or more	School	+	*	Yes
Percentage still enrolled at end of year—90–94%	School	No	No	No
Percentage still enrolled at end of year—95–97%	School	No	No	No
Percentage still enrolled at end of year—98% or higher	School	No	No	No
Student/FTE ratio	School	No	No	No
Percentage of students who are held back and repeating—1–2%	School	*	No	No
Percentage of students who are held back and repeating—3% or more	School	+	No	No
Percentage of teachers absent on average day—3–100%	School	No	No	No
Computers not available in school	Teacher	No	No	No
Extent computers are available to students	Derived from teacher[1]	*	No	No
Number of students in this class—0–20	Teacher	No	No	No
Number of students in this class—21–25	Teacher	No	No	No
Resources provided by school system for math—All	Teacher	No	No	No
Resources provided by school system for math—Most	Teacher	No	No	No

* Statistically significant ($p < .05$).
+ Statistically significant ($p < .10$).
[1] Seven teacher questionnaire items were aggregated to create this variable.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2011 Mathematics Grade 8 Assessment.

Estimation for Figures 6–10 of *School Composition and the Black–White Achievement Gap Report*

Calculation of Summary Data Points

The data points for Black and White student estimated achievement that are displayed in figures 6–10 in the *School Composition and Black–White Achievement Gap* report were calculated from final model regression results, as described here. The data points in the display are

- Average White achievement in each density category
- Average Black achievement in each density category
- Black–White achievement gap—defined as the achievement of Black students in that density category minus the achievement of White students in that density category—which was computed in each density category.

The regression model estimated was

$$Y_i = \beta_0 + \sum_{k=1}^{K}\left\{\beta_k X_{i,k}\right\} + \gamma_0 BLACK_i + \sum_{m=K-M+1}^{K}\left\{\gamma_m X_{i,m} BLACK_i\right\} + \sum_{d=2}^{4}\left\{\delta_d D_{i,d} + \theta_d D_{i,d} BLACK_i\right\} + \varepsilon_i \quad (1)$$

Where

- Y_i is the NAEP Grade 8 mathematics achievement for student i
- $X_{i,k}$ is a set of k independent variables for student i
- $BLACK_i$ is an indicator that = 1 if student i was Black and = 0 otherwise
- $D_{i,d}$ is an indicator that = 1 if the student was in a school in density category d and = 0 otherwise[18]
- k indexes independent variables other than the Black student indicator and density categories
- m indexes independent variables that were interacted with the Black student indicator (M is the total number of independent variables, out of K independent variables that were interacted).

In estimating this equation, we assumed that there was a linear relationship between independent and dependent variables, and that errors were independent, homoscedastic, and normally distributed.[19] The estimated parameters from Equation 1 ($\widehat{\beta_0}$, $\widehat{\beta_k}$, $\widehat{\gamma_0}$, $\widehat{\gamma_m}$, $\widehat{\delta_d}$, and $\widehat{\theta_d}$) and the overall sample average (overall Black and White students) for each independent variable were used together to calculate each data point, using the formulas below. When X_k is an indicator variable, $\overline{X_k}$ is the proportion of the sample when X_k = 1.

White achievement in each density category

The estimated average achievement for White students in the default density category (0–20% Black) holding covariates constant, $\widehat{Y_W^1}$, was calculated by evaluating the estimated regression equation with (a) setting the "Black" indicator to 0, (b) setting the 20–40%, the 40–60%, and the 60–100% density category indicators to 0; and (c) evaluating other covariates at the sample means:

$$\widehat{Y_W^1} = \widehat{\beta_0} + \sum_{k=1}^{K}\widehat{\beta_k}\,\overline{X_k} \quad (2)$$

The estimated average achievement for White students in other density categories ($\widehat{Y_W^2}$ for density 20–40%, $\widehat{Y_W^3}$ for density 40–60%, and $\widehat{Y_W^4}$ for density 60–100%) was estimated by adding the corresponding estimated coefficients for the indicators for those categories ($\widehat{\delta_d}$):

$$\widehat{Y_W^2} = \widehat{\beta_0} + \sum_{k=1}^{K}\left\{\widehat{\beta_k}\,\overline{X_k}\right\} + \widehat{\delta_2} = \widehat{Y_W^1} + \widehat{\delta_2} \quad (3)$$

$$\widehat{Y_W^3} = \widehat{\beta_0} + \sum_{k=1}^{K}\left\{\widehat{\beta_k}\,\overline{X_k}\right\} + \widehat{\delta_3} = \widehat{Y_W^1} + \widehat{\delta_3} \quad (4)$$

$$\widehat{Y_W^4} = \widehat{\beta_0} + \sum_{k=1}^{K}\left\{\widehat{\beta_k}\,\overline{X_k}\right\} + \widehat{\delta_4} = \widehat{Y_W^1} + \widehat{\delta_4} \quad (5)$$

[18] $D_{i,1}$ is an indicator for schools that were 0– 20% Black; $D_{i,2}$ is an indicator for schools that were 20–40% Black; $D_{i,3}$ is an indicator for schools that were 40–60% Black; and $D_{i,4}$ is an indicator for schools that were 60–100% Black.

[19] Regression diagnostics such as scatterplots of residuals versus independent variables, histogram for residuals, plot for residuals, and plot of standardized residuals versus predicted scores did not show any indications that assumptions were not satisfied.

Black achievement in each density category
The estimated average achievement for Black students in other density categories was estimated by setting the Black student indicator, $BLACK_i$, equal to 1 and using similar equations, again using parameters estimated in Equation 1 ($\widehat{\beta_0}$, $\widehat{\beta_k}$, $\widehat{\gamma_0}$, $\widehat{\gamma_m}$, $\widehat{\delta_d}$, and $\widehat{\theta_d}$) and sample means ($\overline{X_k}$).

For Density Category 1, 0–20% Black, the formula was

$$\widehat{Y_B^1} = \widehat{\beta_0} + \sum_{k=1}^{K}\{\widehat{\beta_k}\,\overline{X_k}\} + \widehat{\gamma_0} + \sum_{m=K-M+1}^{K}\left\{\widehat{\gamma_m}\,\overline{X_k}\right\} = \widehat{Y_W^1} + \widehat{\gamma_0} + \sum_{m=K-M+1}^{K}\left\{\widehat{\gamma_m}\,\overline{X_k}\right\} \tag{6}$$

For the other density categories, the formulas were

$$\widehat{Y_B^2} = \widehat{\beta_0} + \sum_{k=1}^{K}\left\{\widehat{\beta_k}\,\overline{X_k}\right\} + \widehat{\gamma_0} + \sum_{m=K-M+1}^{K}\left\{\widehat{\gamma_m}\,\overline{X_k}\right\} + \widehat{\delta_2} + \widehat{\theta_2} = \widehat{Y_B^1} + \widehat{\delta_2} + \widehat{\theta_2} \tag{7}$$

$$\widehat{Y_B^3} = \widehat{\beta_0} + \sum_{k=1}^{K}\left\{\widehat{\beta_k}\,\overline{X_k}\right\} + \widehat{\gamma_0} + \sum_{m=K-M+1}^{K}\left\{\widehat{\gamma_m}\,\overline{X_k}\right\} + \widehat{\delta_3} + \widehat{\theta_3} = \widehat{Y_B^1} + \widehat{\delta_3} + \widehat{\theta_3} \tag{8}$$

$$\widehat{Y_B^4} = \widehat{\beta_0} + \sum_{k=1}^{K}\left\{\widehat{\beta_k}\,\overline{X_k}\right\} + \widehat{\gamma_0} + \sum_{m=K-M+1}^{K}\left\{\widehat{\gamma_m}\,\overline{X_k}\right\} + \widehat{\delta_4} + \widehat{\theta_4} = \widehat{Y_B^1} + \widehat{\delta_4} + \widehat{\theta_4} \tag{9}$$

Achievement gaps in each density category
Achievement gaps were calculated by taking the difference between the two estimated numbers in each density category, again using parameters estimated in Equation 1 ($\widehat{\beta_0}$, $\widehat{\beta_k}$, $\widehat{\gamma_0}$, $\widehat{\gamma_m}$, $\widehat{\delta_d}$, and $\widehat{\theta_d}$) and sample means ($\overline{X_k}$).[20] For Density Category 1, 0–20% Black, this reduced to

$$\widehat{Gap^1} = \widehat{Y_B^1} - \widehat{Y_W^1} = \left\{\widehat{Y_W^1} + \widehat{\gamma_0} + \sum_{m=K-M+1}^{K}\left\{\widehat{\gamma_m}\,\overline{X_k}\right\}\right\} - \left\{\widehat{Y_W^1}\right\} = \widehat{\gamma_0} + \sum_{m=K-M+1}^{K}\left\{\widehat{\gamma_m}\,\overline{X_k}\right\} \tag{10}$$

For the other density categories, the formulas were

$$\widehat{Gap^2} = \widehat{Y_B^2} - \widehat{Y_W^2} = \left\{\widehat{Y_B^1} + \widehat{\delta_2} + \widehat{\theta_2}\right\} - \left\{\widehat{Y_W^1} + \widehat{\delta_2}\right\} = \widehat{Gap^1} + \widehat{\theta_2} \tag{11}$$

$$\widehat{Gap^3} = \widehat{Y_B^3} - \widehat{Y_W^3} = \left\{\widehat{Y_B^1} + \widehat{\delta_3} + \widehat{\theta_3}\right\} - \left\{\widehat{Y_W^1} + \widehat{\delta_3}\right\} = \widehat{Gap^1} + \widehat{\theta_3} \tag{12}$$

$$\widehat{Gap^4} = \widehat{Y_B^4} - \widehat{Y_W^4} = \left\{\widehat{Y_B^1} + \widehat{\delta_4} + \widehat{\theta_4}\right\} - \left\{\widehat{Y_W^1} + \widehat{\delta_4}\right\} = \widehat{Gap^1} + \widehat{\theta_4} \tag{13}$$

Comparisons and Significance Testing

In the report, the following comparisons were made:

- Level of student achievement in higher density categories compared with achievement in the lowest density (0–20% Black) category

[20] The achievement gaps reported are estimated using all students in national public schools. They are calculated as the difference between the average achievement for Black and White students within each density category. Therefore, in the highest density category (60-100 percent Black) the gap is the difference between the averages for all Black students and for all White students that attend such schools, including all Black students who are in schools that are 100 percent Black.

- Achievement gap in higher density categories compared with achievement gap in the lowest categories
- Size of achievement gap in a given category using a model with controls compared with the size of the achievement gap in that same category in the model that did not include control variables.

The first two sets of comparisons were made using two different models: with control variables and without control variables. The models without control variables were effectively equivalent to computing sample means, and because they did not involve any interaction of the percentage-Black variable with other variables, we used sample means to compute the student achievement and achievement gap in these cases.

Level of student achievement in higher density categories compared with achievement in the lowest category

Without controls

For the first set of comparisons of student achievement (i.e., comparisons of estimates in higher density categories with the estimate for the lowest category) *without controls*, we used the jackknife procedure[21] to compute the estimates, the standard errors, and the degrees of freedom. Then, with these statistics, we used *t* tests to compare the level of student achievement in higher density categories with student achievement in the lowest category. The *p* values for these comparisons were adjusted for multiple comparisons for which the race and percentage-Black categories were counted toward family size.

With controls

For the first set of comparisons (i.e., comparisons of estimates in higher density categories with the estimate for the lowest category) *with controls*, the estimates and standard errors came from direct estimation results from AM, using a Black-only or White-only sample. In these models, the lowest category was the reference group (omitted category) for the other percentage-Black categories. Therefore, the estimates and their associated standard errors, *t* statistics, and *p* values obtained from the analysis showed the results of the comparison between the level of student achievement in the higher density categories and the level of student achievement in the lowest category.

Achievement gap in higher density categories compared with achievement gap in the lowest category

Without controls

For the comparisons of gaps across percentage-Black categories (i.e., comparisons of estimates in higher density categories with the estimate for the lowest category) *without control variables*, we computed the mean, standard errors, and degrees of freedom for the achievement gap using the jackknife procedure. Then we used these statistics to compare the achievement gap in higher density categories with the achievement gap in the lowest category. The *p* values for these comparisons were adjusted for multiple comparisons for which the Black categories were counted toward family size.

With controls

The estimates and their standard errors for the comparisons *with controls* were generated as part of the AM results. We used the estimates and their standard errors for the interaction between the percentage-Black categories and the Black variable to compare the achievement gap in higher density categories with the achievement gap in the lowest categories.

Size of achievement gap in a given density category using a model with controls compared with the size of the achievement gap in that same category in a model that did not include control variables

To compare the size of achievement gap in a given density category from a model with controls with the achievement gap from a model that did not include control variables, we computed the overall achievement gap in the model with controls, using the parameter estimates. Then we computed the standard error for this predicted gap, using the standard error of the estimates used in the computation and the variance–covariance

[21] Jackknife procedure is described in detail at http://nces.ed.gov/nationsreportcard/tdw/weighting/2002_2003/weighting_2003_repwts_appdx.aspx.

matrix from the direct estimation results. We compared the size of the achievement gap with controls with the size of achievement gap without controls, using the respective estimates and their respective standard errors.[22]

Results

Table 6 shows the results for the regression model (1) using direct estimation. The results show that, in addition to the main effect of Density Category 3 (school percentage Black 40–60%), there were significant interaction effects between the Black student indicator and Density Category 3 (school percentage Black 40–60%) and Density Category 4 (school percentage Black 60–100%). Black students who were in schools that were 40–60% Black were found to be performing an additional 4.07 NAEP mathematics points lower than their counterparts in schools that were 0–20% Black. Black students in schools that were 60–100% Black were found to be performing an additional 5.32 points lower than their counterparts in schools that were 0–20% Black. The results also showed that in addition to the main effect of the student/FTE ratio, there was a significant interaction between the student/FTE ratio and the Black student indicator; when the ratio was higher by one student per FTE, Black student scores were an additional 0.38 points lower than those of their counterparts.

Tables 7 and 8 display the separate regression results for female and male samples. The results showed that for female students the main effects of the density variables and their interaction with the Black student indicator were not statistically significant. However, results for male students followed a pattern similar to the overall results. Black male students who were in schools that were 40–60% Black and in schools 60–100% Black were found to be performing 5.04 and 8.79 points, respectively, lower than White male students in schools that were 0–20% Black.

[22] The standard errors for the model without controls were computed using the standard NAEP jackknife procedure, whereas the standard errors for the model with controls were computed using Taylor series estimation. To test whether there was a change in the significance of comparison of the gap between the model with controls and the model without controls, we computed Taylor series standard errors for the model without controls and used these instead of jackknife standard errors. There were no meaningful differences between the results using jackknife standard errors and the results using Taylor series estimation of standard errors.

Table 6. Regression results for the relationship between NAEP scale scores and density, controlling for student, teacher, and school characteristics

Parameter	Estimate	Standard error	*t*-Stat	$p > \|t\|$
Constant	278.86	4.83	-1.14	0.26
Black	-4.65	2.52	-1.84	0.07
Male	3.57*	0.39	9.07	0.00
Student with mother's highest education unknown	0.20	0.84	0.24	0.81
Student with mother's highest education ≥ HS	4.03*	0.73	5.54	0.00
Student with mother's highest education ≥ BA	6.67*	0.78	8.58	0.00
Student with father's highest education unknown	2.77*	0.94	2.93	0.01
Student with father's highest education ≥ HS	4.42*	0.86	5.12	0.00
Student with father's highest education ≥ BA	9.48*	0.85	11.11	0.00
Eligible for National School Lunch Program = Yes	-6.84*	0.47	-14.64	0.00
IEP status = Yes	-27.86*	0.76	-36.65	0.00
Student's home has ≥ 26 books	13.18*	0.47	27.97	0.00
Student's home has an encyclopedia = No	-1.61*	0.60	-2.70	0.01
Student's home has an encyclopedia = missing value	-1.29*	0.53	-2.45	0.02
Grade 8: Weighted proportion male	-5.07*	2.04	-2.49	0.02
Grade 8: Weighted proportion of students with parents' highest education unknown	3.39	5.34	0.64	0.53
Grade 8: Weighted proportion of students with parents' highest education ≥ HS	-6.81	4.50	-1.51	0.13
Grade 8: Weighted proportion of students with parents' highest education ≥ BA	6.59	4.30	1.53	0.13
School-level weighted proportion eligible for National School Lunch Program	-8.36*	2.16	-3.87	0.00
School-level weighted proportion IEP status = Yes	-3.17	2.88	-1.10	0.28
School-level weighted proportion of students with ≥ 26 books at home	5.87*	2.49	2.36	0.02
School-level weighted proportion of students with encyclopedia = No	-7.11	3.94	-1.80	0.08
School-level weighted proportion of students with encyclopedia = Missing value	-1.62	2.47	-0.66	0.51
Density Category 2: School-level proportion Black is 20–40%	0.86	0.84	1.02	0.31
Density Category 3: School-level proportion Black is 40–60%	3.88*	1.36	2.86	0.01
Density Category 4: School-level proportion Black is 60–100%	1.89	1.99	0.95	0.35
Teacher's degree (undergraduate or graduate) = Math major	3.97*	0.78	5.10	0.00

See notes at end of table.

Table 6. Regression results for the relationship between NAEP scale scores and density, controlling for student, teacher, and school characteristics—Continued

Parameter	Estimate	Standard error	*t*-Stat	$p > \|t\|$
Teacher's degree (undergraduate or graduate) = Math minor	2.10*	0.88	2.38	0.02
Teacher uses different set of methods to teach some students = Large extent	-3.09*	0.73	-4.25	0.00
Teacher uses different set of methods to teach some students = Moderate extent	-1.56*	0.58	-2.68	0.01
Teacher assigns math homework per day ≥ 1 hour	3.28*	0.73	4.48	0.00
School-level percentage of students absent on average day = 3–5	0.01	0.73	0.02	0.99
School-level percentage of students absent on average day = 6–100	-2.61*	0.96	-2.71	0.01
Student/full time-employee (FTE) ratio	-0.18*	0.05	-3.74	0.00
(Density Category 2: School-level proportion Black is 20–40%) × Black	-1.31	1.44	-0.91	0.37
(Density Category 3: School-level proportion Black is 40–60%) × Black	-4.07*	1.89	-2.16	0.03
(Density Category 4: School-level proportion Black is 60–100%) × Black	-5.32*	2.06	-2.59	0.01
(Teacher's degree [undergraduate or graduate] = math major) × Black	-2.06	1.32	-1.56	0.12
(Teacher's degree [undergraduate or graduate] = math minor) × Black	-2.38	1.62	-1.47	0.15
(Teacher uses different methods to teach some students = Large extent) × Black	-1.09	1.33	-0.82	0.41
(Teacher uses different methods to teach some students = Moderate extent) × Black	-1.35	1.16	-1.17	0.25
(Teacher assigns math homework per day ≥ 1 hour) × Black	-2.44	1.46	-1.67	0.10
(School-level percentage of students absent on average day = 3–5) × Black	-1.07	1.41	-0.76	0.45
(School-level percentage of students absent on average day = 6–100) × Black	-2.55	1.72	-1.48	0.14
(Student/FTE ratio) × Black	-0.38*	0.10	-3.80	0.00
(IEP status = Yes) × Black	-2.52	1.70	-1.48	0.14

* Statistically significant ($p < .05$).
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2011 Mathematics Grade 8 Assessment.

Table 7. Regression results for the relationship between NAEP scale scores and density, controlling for student, teacher, and school characteristics, for female students

Parameter	Estimate	Standard error	*t*-Stat	$p > \|t\|$
Constant	280.53	6.21	-0.62	0.54
Black	-2.89	3.45	-0.84	0.41
Student with mother's highest education unknown	0.68	1.25	0.54	0.59
Student with mother's highest education ≥ HS	5.58*	1.05	5.32	0.00
Student with mother's highest education ≥ BA	8.38*	1.06	7.91	0.00
Student with father's highest education unknown	1.60	1.29	1.24	0.22
Student with father's highest education ≥ HS	4.35*	1.15	3.78	0.00
Student with father's highest education ≥ BA	9.63*	1.22	7.91	0.00
Eligible for National School Lunch Program = Yes	-6.76*	0.66	-10.29	0.00
IEP status = Yes	-31.97*	1.24	-25.80	0.00
Student's home has ≥ 26 books	12.86*	0.67	19.24	0.00
Student's home has an encyclopedia = No	-0.82	0.80	-1.02	0.31
Student's home has an encyclopedia = Missing value	-0.82	0.69	-1.20	0.24
Grade 8: Weighted proportion male	-4.47	2.76	-1.62	0.11
Grade 8: Weighted proportion of students with parents' highest education unknown	0.83	6.89	0.12	0.91
Grade 8: Weighted proportion of students with parents' highest education ≥ HS	-10.93	5.85	-1.87	0.07
Grade 8: Weighted proportion of students with parents' highest education ≥ BA	3.89	5.75	0.68	0.50
School-level weighted proportion eligible for National School Lunch Program	-7.01*	2.97	-2.36	0.02
School-level weighted proportion IEP status = Yes	-3.37	3.59	-0.94	0.35
School-level weighted proportion of students with ≥ 26 books at home	5.22	2.99	1.75	0.09
School-level weighted proportion of students with encyclopedia = No	-8.17	4.78	-1.71	0.09
School-level weighted proportion of students with encyclopedia = Missing value	-1.46	3.43	-0.43	0.67
Density Category 2: School-level percentage Black is 20–40%	0.22	1.18	0.18	0.86
Density Category 3: School-level percentage Black is 40–60%	3.24	2.03	1.60	0.12
Density Category 4: School-level percentage Black is 60–100%	-0.92	2.76	-0.33	0.74
Teacher's degree (undergraduate or graduate) = Math major	3.66*	1.00	3.67	0.00
Teacher's degree (undergraduate or graduate) = Math minor	1.47	1.07	1.38	0.17

See notes at end of table.

Table 7. Regression results for the relationship between NAEP scale scores and density, controlling for student, teacher, and school characteristics, for female students—Continued

Parameter	Estimate	Standard error	*t*-Stat	$p > \|t\|$
Teacher uses different set of methods to teach some students = Large extent	-3.45*	0.87	-3.98	0.00
Teacher uses different set of methods to teach some students = Moderate extent	-2.10*	0.78	-2.70	0.01
Teacher assigns math homework per day ≥ 1 hour	3.36*	0.90	3.73	0.00
School-level percentage of students absent on average day = 3–5	0.53	0.84	0.63	0.53
School-level percentage of students absent on average day = 6–100	-1.75	1.10	-1.58	0.12
Student/FTE ratio	-0.18*	0.06	-3.17	0.00
(Density Category 2: School-level percentage Black is 20–40%) × Black	0.34	2.04	0.17	0.87
(Density Category 3: School-level percentage Black is 40–60%) × Black	-3.17	2.66	-1.19	0.24
(Density Category 4: School-level percentage Black is 60–100%) × Black	-1.67	2.89	-0.58	0.57
(Teacher's degree [undergraduate or graduate] = Math major) × Black	-1.92	1.74	-1.10	0.28
(Teacher's degree [undergraduate or graduate] = Math minor) × Black	-2.09	1.99	-1.05	0.30
(Teacher uses different methods to teach some students = Large extent) × Black	0.06	1.79	0.03	0.98
(Teacher uses different methods to teach some students = Moderate extent) × Black	-0.76	1.51	-0.51	0.62
(Teacher assigns math homework per day ≥ 1 hour) × Black	-3.18	2.03	-1.56	0.12
(School-level percentage of students absent on average day = 3–5) × Black	-2.70	1.77	-1.53	0.13
(School-level percentage of students absent on average day = 6–100) × Black	-5.41*	2.07	-2.61	0.01
(Student/FTE ratio) × Black	-0.33*	0.15	-2.22	0.03
(IEP status = Yes) × Black	-0.34	2.81	-0.12	0.90

* Statistically significant ($p < .05$).
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2011 Mathematics Grade 8 Assessment.

Table 8. Regression results for the relationship between NAEP scale scores and density, controlling for student, teacher, and school characteristics, for male students

Parameter	Estimate	Standard error	*t*-Stat	$p > \|t\|$
Constant	281.81	6.36	-0.41	0.69
Black	-6.41*	3.33	-1.92	0.06
Student with mother's highest education unknown	-1.45	1.38	-1.06	0.30
Student with mother's highest education ≥ HS	1.76	1.21	1.46	0.15
Student with mother's highest education ≥ BA	4.32*	1.27	3.39	0.00
Student with father's highest education unknown	4.20*	1.22	3.44	0.00
Student with father's highest education ≥ HS	4.71*	1.11	4.24	0.00
Student with father's highest education ≥ BA	9.57*	1.16	8.25	0.00
Eligible for National School Lunch Program = Yes	-6.79*	0.71	-9.63	0.00
IEP status = Yes	-26.02*	0.94	-27.56	0.00
Student's home has ≥ 26 books	13.51*	0.63	21.61	0.00
Student's home has an encyclopedia = No	-2.40*	0.81	-2.96	0.00
Student's home has an encyclopedia = Missing value	-1.69*	0.80	-2.12	0.04
Grade 8: Weighted proportion male	-5.68	2.87	-1.98	0.05
Grade 8: Weighted proportion of students with parents' highest education unknown	5.76	6.69	0.86	0.39
Grade 8: Weighted proportion of students with parents' highest education ≥ HS	-2.83	5.67	-0.50	0.62
Grade 8: Weighted proportion of students with parents' highest education ≥ BA	9.02	5.65	1.60	0.12
School-level weighted proportion eligible for National School Lunch Program	-9.99*	2.37	-4.22	0.00
School-level weighted proportion IEP status = Yes	-2.62	3.98	-0.66	0.51
School-level weighted proportion of students with ≥ 26 books at home	6.19*	3.03	2.04	0.05
School-level weighted proportion of students with encyclopedia = No	-5.97	4.58	-1.30	0.20
School-level weighted proportion of students with encyclopedia = Missing value	-1.80	3.37	-0.54	0.59
Density Category 2: School-level percentage Black is 20–40%	1.58	1.02	1.55	0.13
Density Category 3: School-level percentage Black is 40–60%	4.41*	1.51	2.92	0.01
Density Category 4: School-level percentage Black is 60–100%	4.56	2.90	1.57	0.12
Teacher's degree (undergraduate or graduate) = Math major	4.24*	0.97	4.39	0.00
Teacher's degree (undergraduate or graduate) = Math minor	2.70*	1.09	2.47	0.02

See notes at end of table.

Table 8. Regression results for the relationship between NAEP scale scores and density, controlling for student, teacher, and school characteristics, for male students—Continued

Parameter	Estimate	Standard error	*t*-Stat	$p > \|t\|$
Teacher uses different set of methods to teach some students = Large extent	-2.75*	0.95	-2.89	0.01
Teacher uses different set of methods to teach some students = Moderate extent	-1.04	0.72	-1.44	0.15
Teacher assigns math homework per day ≥ 1 hour	3.22*	1.02	3.17	0.00
School-level percentage of students absent on average day = 3–5	-0.52	0.90	-0.58	0.56
School-level percentage of students absent on average day = 6–100	-3.42*	1.20	-2.86	0.01
Student/FTE ratio	-0.19*	0.06	-3.11	0.00
(Density Category 2: School-level percentage Black is 20–40%) × Black	-3.04	1.93	-1.57	0.12
(Density Category 3: School-level percentage Black is 40–60%) × Black	-5.04*	2.20	-2.29	0.03
(Density Category 4: School-level percentage Black is 60–100%) × Black	-8.79*	3.19	-2.76	0.01
(Teacher's degree [undergraduate or graduate] = Math major) × Black	-2.30	1.75	-1.32	0.19
(Teacher's degree [undergraduate or graduate] = Math minor) × Black	-2.50	2.29	-1.09	0.28
(Teacher uses different methods to teach some students = Large extent) × Black	-2.09	1.99	-1.05	0.30
(Teacher uses different methods to teach some students = Moderate extent) × Black	-1.84	1.63	-1.13	0.26
(Teacher assigns math homework per day ≥ 1 hour) × Black	-1.71	1.90	-0.90	0.37
(School-level percentage of students absent on average day = 3–5) × Black	0.32	1.90	0.17	0.87
(School-level percentage of students absent on average day = 6–100) × Black	0.27	2.36	0.11	0.91
(Student/FTE ratio) × Black	-0.43*	0.13	-3.20	0.00
(IEP status = Yes) × Black	-2.36	2.15	-1.10	0.28

* Statistically significant ($p < .05$).
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2011 Mathematics Grade 8 Assessment.

Methodology for "Exploring 'Between-School' and 'Within-School' Achievement Gaps" Chapter

The decomposition analysis is a descriptive analysis that examines whether Black students are more likely than White students to attend schools associated with lower achievement (i.e. the gaps are largely "between" schools) or whether the achievement gaps are largely between Black and White students within the same schools (i.e., the gaps are "within" schools). The analyses conducted for this study do not offer suggestions for how to reduce gaps between or within schools.[23]

The methodology used to decompose achievement gaps, developed by Reardon (2008), actually focuses on three components: between-school, within-school, and ambiguous. In this analysis, the "ambiguous" component is renamed "indeterminate." The details of this methodology and its implementation are described below. Similar to the previous analyses, decomposition analyses used individual level data but instead of examining achievement in various density categories, density (i.e., percent of students in the school who were Black) was used as a continuous variable in these analyses.

To calculate the three achievement gap components (between-school, within-school, and ambiguous), the study estimated the following:

1. **Holding Black student density constant, the analysis estimated the association between a student's race (i.e., being a Black/African-American student) and achievement.** This relationship is the primary determinant of the "within-school" component of the achievement gap as it measures achievement as associated with a student's race regardless of the school the student attends.[24]

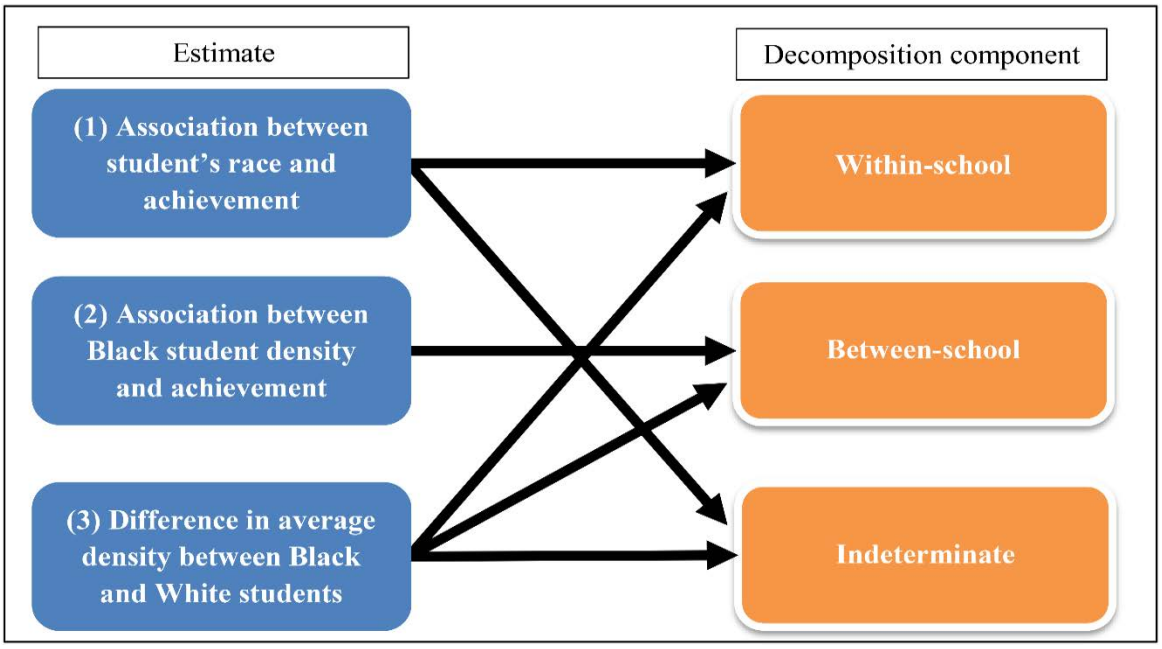
2. **Holding a student's race constant, the analysis estimated the association between Black student density and achievement.** This relationship is the primary determinant of the "between-school" component as it measures how achievement varies when the density of Black students in a student's school varies.

3. **The analysis estimated the difference in average density between Black and White students.** This estimate provides a "weight" that shifts the relative importance of components #1 and #2, that is, the allocation of the gap to the within-school versus between-school components. In particular, the greater the difference in average density between the schools that Black and White students attend, the greater the emphasis placed on component #2 and the greater the portion of the gap that is attributed to between-school differences. By contrast, the smaller the difference in density between the schools that Black and White students attend, the greater the emphasis placed on component #1 and the greater the portion of the gap that is attributed to within-school differences.

The use of each of these pieces of information in calculating each component of the decomposition is illustrated in figure 1.

[23] This analysis does not examine what causes or contributes to achievement gaps. Hence, while policies may be used to reduce between and within schools gaps by reallocating resources between and/or within schools, this analysis does not indicate whether or not, or the extent to which, such policies might or might not be contributing to the gaps.
[24] The NAEP sampling frame does not support the comparisons of individual students; hence, students observed in a given school cannot be compared to each other. The information necessary for the decomposition, however, is obtained by comparing students across all schools as described here.

Figure 1. The relation between different estimates and the resulting outcomes for the decomposition of the Black-White achievement gap.



The first two estimates shown in figure 1, estimates #1 and #2, were derived using a regression equation that was estimated for public schools nationally, as a whole, and for each jurisdiction separately. The regression equation used the following specification, with *i* indexing students and *s* indexing schools.[25]

$$Y_{is} = \beta_0 + \beta_1 Black_i + \gamma_1 PctBlack_s + \delta_1 Hispanic_i + \delta_2 Asian_i + \delta_3 AmericanIndian_i + \varepsilon_{is} \quad (14)$$

Where

- Y_{is} = NAEP achievement for mathematics Grade 8 for student i in school s
- $Black_i$ = student-level race indicator that = 1 if student i was Black and = 0 otherwise
- $PctBlack_s$ = percentage of students in the school s who were Black
- $Hispanic_i$ = student-level race indicator that = 1 if student i was Hispanic and = 0 otherwise
- $Asian_i$ = student-level race indicator that = 1 if student i was Asian/Pacific Islander and = 0 otherwise
- $AmericanIndian_i$ = student-level race indicator that = 1 if student i was American Indian and = 0 otherwise.

The third estimate listed above in figure 1, the difference in average density, was derived by calculating the average density separately for Black students and White students in the public schools nationally and then for each jurisdiction (i.e., each state and the District of Columbia) separately. To simplify the discussion, the notation from Page, Murnane, and Willett (2008) is used where $\hat{V}$ is defined as the difference in average density between Black and White students.

[25] Because all students are included in this analysis regardless of race or ethnicity, the estimated association between Black student density and achievement (γ_1) is an average relationship across all students.

$$\hat{V} = \overline{PctBlack}^{Black} - \overline{PctBlack}^{White} \quad (15)$$

Where

- $\overline{PctBlack}^{Black}$ is the average percentage of students in the school who were Black for Black students
- $\overline{PctBlack}^{White}$ is the average percentage of students in the school who were Black for White students.

As an example, nationally, in the 2011 NAEP Grade 8 mathematics sample, Black students on average attended schools that were 48% Black (i.e., $\overline{PctBlack}^{Black}$ = 48%), and White students on average attended schools that were 9% Black (i.e., $\overline{PctBlack}^{White}$ = 9%); so the difference in density in public schools nationally was equal to 48 – 9 = 39 percentage points.

In addition, let $\widehat{\beta}_1$ and $\widehat{\gamma}_1$ be the parameters estimated from Equation 14. With these terms defined, the decomposition equations used to determine the portion of the Black–White achievement gap attributable to between-school, within-school, and indeterminate components are, as developed by Reardon (2008), the following:

- Within-school gap = $\widehat{\beta}_1\left(1-\hat{V}\right)$
- Between-school gap = $\widehat{\gamma}_1\hat{V}$
- Indeterminate school gap = $\widehat{\beta}_1\hat{V}$.

The *within-school gap* is an interaction between the estimated difference in achievement between Black and White students, $\widehat{\beta}_1$, and 1 minus the difference in the average density (i.e., percentage of Black students in the school) between Black and White students, $(1 - \hat{V})$. On the other hand, the *between-school gap* is the interaction between the estimated relationship between the proportion of Black students in a school and student achievement, $\widehat{\gamma}_1$, and the difference in the average density between Black and White students, $\hat{V}$. Similarly, the *indeterminate school gap* is the interaction between the estimated difference in achievement between Black and White students, $\widehat{\beta}_1$, and the difference in the average density between Black and White students, $\hat{V}$.

Why are there no student or school control variables in the regression model?

The absence of student and school control variables in the regression model may cause confusion as it is a departure from the analysis presented in the first chapter of the report on the relationship between achievement and density where SES and other factors were disregarded. The research in the previous chapter sought to investigate whether density was correlated with the achievement gap. In such an investigation, one would want to control for SES and other potentially confounding factors to examine potential relationships between density, the variable of interest, and the achievement gap. The decomposition analysis is different in that it is descriptive and not seeking to determine a correlational relationship. Specifically, the decomposition analysis is a description of where the achievement gaps are occurring so that policies might be optimally directed (e.g., focused on the distribution of resource within, rather than between, schools).

Comparisons and Significance Testing

In the report, the following comparisons were made in figure 13:

- The size of the within-school achievement gap with that of the between-school achievement gap
- The size of the indeterminate achievement gap with that of the between-school achievement gap.

The estimates and standard errors for the three components of the achievement gap (within, between, and indeterminate) were computed using the jackknife procedure. We also computed the estimates of the within- and between-school achievement gap comparison and indeterminate and between-school gap comparison by jackknife procedure. We used *t* tests to compute the significance of these comparisons and applied multiple comparisons corrections for the resulting *p* values by counting the type of achievement gap (within, between, and indeterminate) toward the family size (i.e., as elements in the domain within which we were defining multiple comparisons).

Results

Table 9 displays the regression coefficients from the decomposition analyses that were estimated separately for each jurisdiction that had a sufficient number of Black students to carry out the analysis. In addition, the table displays the mean school proportion Black for Black students, the mean school proportion Black for White students, and the difference between the two means. To compute the three components of the decomposition, we used the difference between the two means, along with the regression coefficients on the Black indicator and on proportion Black. For example, at the national level, multiplying the difference in densities (.39) by the proportion Black coefficient (−11.67) estimated the between-school gap to be 4.6.

Table 9. Results of the decomposition analysis, by jurisdiction

Jurisdiction	Mean proportion Black for Black students	Mean proportion Black for White students	Difference	Regression coefficients					
				Intercept	Black	Proportion Black	Hispanic	American Indian/Alaska Native	Asian/Pacific Islander
Nation	0.48	0.09	0.39	293.58	-26.02	-11.67	-22.80	-26.85	10.11
AK	0.08	0.04	0.05	296.04	-20.01	3.66	-18.08	-37.88	-10.90
AL	0.63	0.19	0.44	281.07	-28.17	-5.57	-24.52	-1.72	14.67
AR	0.56	0.12	0.44	289.31	-22.44	-18.22	-15.12	-19.59	-1.07
AZ	0.09	0.05	0.04	293.37	-25.90	5.76	-28.02	-40.99	10.96
CA	0.24	0.05	0.20	291.28	-30.81	-28.64	-29.68	-24.25	9.04
CO	0.21	0.04	0.17	302.37	-30.44	-9.71	-30.75	-35.97	11.13
CT	0.32	0.07	0.25	299.53	-24.78	-39.65	-28.11	-20.72	11.38
DC	0.90	0.51	0.39	343.13	-52.85	-37.88	-54.90	0.00	-21.43
DE	0.41	0.30	0.12	302.34	-24.94	-30.40	-17.70	-21.71	18.49
FL	0.42	0.16	0.26	289.77	-25.97	-12.22	-13.77	0.73	24.08
GA	0.61	0.24	0.37	294.37	-24.04	-13.33	-13.54	1.09	10.55
HI	0.07	0.03	0.04	286.91	-16.57	86.52	-27.77	-20.28	-11.38
IA	0.17	0.04	0.13	289.02	-27.93	-20.08	-18.65	-22.84	3.79
ID	0.02	0.01	0.00	288.38	-21.37	188.25	-22.80	-32.55	8.73
IL	0.62	0.06	0.56	293.89	-28.58	-8.57	-21.12	-31.61	20.24
IN	0.54	0.06	0.48	290.02	-24.41	-3.49	-14.26	-2.13	22.38
KS	0.26	0.06	0.20	296.57	-22.13	-20.80	-19.64	-10.99	6.54
KY	0.30	0.08	0.22	284.75	-23.09	-3.34	-15.17	12.14	21.14
LA	0.60	0.27	0.33	284.94	-20.49	-9.05	-12.52	-7.19	17.97
MA	0.30	0.03	0.27	304.50	-27.33	-6.17	-30.91	-20.37	16.11
MD	0.63	0.17	0.46	307.62	-23.69	-25.69	-25.59	-21.92	10.12
ME	0.10	0.02	0.08	289.71	-26.65	1.31	-9.26	-22.36	5.69
MI	0.56	0.06	0.50	286.77	-28.18	-15.60	-10.75	-10.09	25.57
MN	0.29	0.05	0.24	302.36	-34.35	-11.12	-31.65	-38.78	-18.31
MO	0.57	0.07	0.49	288.04	-29.66	-7.02	-19.86	-13.42	13.86

See notes at end of table.

Table 9. Results of the decomposition analysis, by jurisdiction—Continued

Jurisdiction	Mean proportion Black for Black students	Mean proportion Black for White students	Difference	Regression coefficients					
				Intercept	Black	Proportion Black	Hispanic	American Indian/Alaska Native	Asian/Pacific Islander
MS	0.71	0.28	0.43	285.92	-23.10	-10.10	-8.10	-25.21	34.46
MT	0.01	0.01	0.00	296.41	-5.21	29.78	-11.73	-33.00	16.21
NC	0.47	0.20	0.27	298.02	-25.81	-10.10	-19.76	-31.82	18.79
ND	0.04	0.02	0.02	294.98	-35.92	68.27	-18.52	-31.47	-2.21
NE	0.25	0.05	0.20	291.40	-28.99	-32.06	-27.66	-21.70	9.36
NH	0.04	0.02	0.02	296.75	-19.74	-203.81	-20.92	-11.58	11.01
NJ	0.46	0.08	0.37	305.16	-27.90	-12.81	-28.48	0.00	13.65
NM	0.05	0.03	0.02	285.67	-26.12	111.49	-19.19	-29.39	14.53
NV	0.16	0.08	0.08	294.50	-30.81	-28.50	-24.76	-18.68	-3.46
NY	0.49	0.07	0.42	291.69	-19.89	-15.42	-25.16	-34.24	13.30
OH	0.59	0.08	0.51	296.51	-23.68	-17.13	-20.24	10.35	8.12
OK	0.32	0.08	0.25	286.47	-24.13	0.47	-22.53	-13.88	17.43
OR	0.28	0.02	0.26	286.94	-25.04	5.03	-19.66	-26.02	8.87
PA	0.62	0.06	0.55	296.24	-20.95	-29.82	-19.46	25.78	17.51
RI	0.21	0.04	0.16	294.22	-24.90	-57.29	-23.43	-36.54	-0.47
SC	0.53	0.29	0.23	296.56	-27.38	-13.36	-19.03	-6.03	18.22
SD	0.05	0.02	0.03	295.93	-24.59	-22.80	-21.06	-32.45	-10.27
TN	0.59	0.08	0.51	281.49	-21.95	-10.41	-13.40	19.89	29.22
TX	0.30	0.12	0.19	304.16	-25.10	-6.91	-20.53	-7.38	13.13
UT	0.02	0.01	0.00	294.46	-29.74	-368.01	-28.77	-44.46	-3.26
VA	0.45	0.17	0.27	299.54	-24.68	-15.11	-18.17	-24.66	15.50
VT	0.05	0.01	0.04	295.36	-27.83	-61.22	-6.21	-39.70	19.95
WA	0.12	0.04	0.08	296.54	-25.90	-51.00	-24.75	-38.44	12.42
WI	0.52	0.04	0.48	295.43	-32.53	-15.45	-23.89	-35.38	-4.35
WV	0.17	0.05	0.12	273.45	-15.64	12.14	-14.84	20.45	33.52
WY	0.03	0.01	0.01	292.75	-21.24	-105.02	-19.31	-34.22	4.11

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2011 Mathematics Grade 8 Assessment.

References

Chapin, F.S. (1933). *The Measurement of Social Status.* Minneapolis: University of Minnesota Press.

Cheema, J.R., and Galuzzo, G. (2013). Analyzing the Gender Gap in Math Achievement: Evidence From a Large-Scale US Sample. *Research in Education, 90:* 98–112.

Harwell, M., and LeBeau, B. (2010). Student Eligibility for a Free Lunch as an SES Measure in Education Research. *Educational Researcher, 39*(2): 120–131.

Konstantopoulos, S., and Hedges, L. (2008). How Large an Effect Can We Expect From School Reforms? *Teachers College Record, 110*(8): 1611–1638

Page, L.C., Murnane, R.J., and Willett, J.B. (2008). *Trends in the Black-White Achievement Gap: Clarifying the Meaning of Within- and Between-School Achievement Gaps* (NBER Working Paper No. 14213). Cambridge, MA: National Bureau of Economic Research.

Reardon, S.F. (2008). *Thirteen Ways of Looking at the Black-White Test Score Gap* (Working Paper No. 2008-08). Palo Alto, CA: Stanford University. Retrieved March 25, 2014, from http://steinhardt.nyu.edu/scmsAdmin/uploads/001/766/24%20reardon%20black-white%20gap%20march%202007.pdf.

Sewell, W.H. (1940). The Construction and Standardization of a Scale for the Measurement of the Socioeconomic Status of Oklahoma Farm Families. In *Agricultural Experiment Station Technical Bulletin 9* (pp. 42–54). Stillwater, OK: Oklahoma Agricultural and Mechanical College.

Sirin, S.R. (2005). Socioeconomic Status and Academic Achievement: A Meta-Analytic Review. *Review of Educational Research, 75*(3): 417–453.

U.S. Department of Education. Institute of Education Sciences, National Center for Education Statistics. (2012). *Improving the Measurement of Socioeconomic Status for the National Assessment of Educational Progress: A Theoretical Foundation.* Retrieved June 3, 2015, from: http://nces.ed.gov/nationsreportcard/researchcenter/socioeconomic_factors.aspx.

www.ed.gov

ies.ed.gov